Junior Wiskunde Olympiade Problems part 1



Saturday 3 June 2023 Vrije Universiteit Amsterdam

- The problems in part 1 are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form.
- A correct answer is awarded 2 points, for a wrong answer no points are deducted.
- You are allowed to use draft paper. The use of compass, ruler or set square is allowed. Calculators and comparable devices are not allowed.
- You have 45 minutes to finish these problems. Good luck!
- 1. How many three-digit numbers, with each digit unequal to zero, are there such that the three digits add up to 7?
 - A) 4 B) 7 C) 10 D) 15 E) 21
- 2. A baker baked a big square cake. He wants to cut the cake. He cuts the cake only lengthwise or widthwise, all the way from one edge to the opposite edge.

At least how many cuts does the baker need in order to cut the cake into exactly 180 pieces?

- A) 25 B) 26 C) 27 D) 28 E) 29
- **3.** On a sheet of paper the numbers 1 to 6 are written as in the figure on the right. The sheet is folded over the dotted lines. This creates a stack of 6 layers, where each layer has a number (that can be written on either side of that layer).

1	2	3
4	5	6

If number 1 is on the top layer, then which number can \underline{not} be on the bottom layer?

- A) 2 B) 3 C) 4 D) 5 E) 6
- 4. A green, a blue and an red dragon all do not like one of the three vegetables leek, spinach and carrot; each a different one. They also all have a favourite vegetable out of these three, again each a different one. They all make two statements.
 - The green dragon says: "My favourite vegetable is leek; the red dragon doesn't like it."
 - The blue dragon says: "I don't like carrots; the green dragon does."
 - The red dragon says: "I do like spinach; leek is the blue dragon's favourite vegetable."

All dragons have made one true and one false statement. Which dragon has which favourite vegetable?

A)	green:	leek,	blue:	spinach,	red:	carrot.
B)	green:	leek,	blue:	carrot,	red:	spinach.
C)	green:	spinach,	blue:	leek,	red:	carrot.
D)	green:	spinach,	blue:	carrot,	red:	leek.
E)	green:	carrot,	blue:	leek,	red:	spinach.

5. A square is divided into nine small squares containing the numbers 1, 2 and 3; see the left figure below. In a move, you turn each 1 in a row or column into a 2, each 2 in that row or column into a 3, and each 3 in that row or column into a 1. First you change a column of your choice, then a row of your choice, then again a column, and so on. In the move below, the first column is changed. Starting from the figure on the left, at some point we want to see only ones on the square.

1	2	3		2	2	3
2	3	1	\longrightarrow	3	3	1
3	1	2		1	1	2

What is the smallest number of moves required to do so?

6. In the figure on the right, you have to fill the little circles with the numbers 1 to 9, using one number twice and the rest only once. For a straight line that has exactly three circles on it and for an arc (dashed) that has three circles on it, the numbers in those three circles must add up to 15. For a straight line containing four circles, the numbers in those four circles must add up to 18.

Which number is used twice?

- A) 2 B) 3 C) 4 D) 5 E) 6
- 7. A rectangle ABCD stands with side AB on the ground and with side BC against the wall. The length of the side AD is 4, the length of the side CD is 3, and the length of the diagonal AC is 5. The rectangle is tilted with point A as the pivot point such that point C is straight above point A. In this situation, what is the distance from point B to the wall?
 - A) $\frac{2}{3}$ B) $\frac{3}{5}$ C) $\frac{5}{8}$ D) $\frac{7}{12}$ E) $\frac{8}{13}$
- 8. Digits can be displayed digitally by turning seven oblong lights (arranged in the shape of an 8) each on or off. Below you can see how all digits from 1 to 9 can be displayed digitally.



Jan has a device that allows him to display one digit at the time. He can manually turn each light on or off by flipping a switch. He now wants to display all digits from 1 to 9 in succession, not necessarily in ascending order, using as few flips of the switches as possible in total. For example, if he goes from 4 to 7, he has to turn 2 lights off and 1 on, so a total of 3 flips. Jan gets to choose which digit to start with; from then on, he starts counting how many times he has to flip a switch.

What is the minimum number of times he has to flip a switch?

A) 9 B) 10 C) 11 D) 12 E) 13



