Junior Wiskunde Olympiade
Problems part 1

Saturday 24 September 2022
Vrije Universiteit Amsterdam

- The problems in part 1 are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form.

- A correct answer is awarded 2 points, for a wrong answer no points are deducted.

- You are allowed to use draft paper. The use of compass, ruler or set square is allowed. Calculators and comparable devices are not allowed.

- You have 45 minutes to finish these problems. **Good luck!**

1. Joah has a very long liquorice lace. He keeps taking bites out of the lace (but not from the very beginning or end of the lace), each time eating 2 cm of the liquorice, creating two smaller pieces in the process. He repeats this several times. At the end, he is left with pieces of liquorice lace of 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 cm.
How long (in cm) was his liquorice lace originally?

   A) 55 B) 66 C) 73 D) 75 E) 81

2. Five distinct positive integers are in a sequence ordered from small to large. The middle number is 20. The difference between the smallest two numbers equals the difference between the largest two numbers. The fourth number is four times as large as the first number, and the fifth number is twice as large as the second number.
When you add all five numbers, what is the outcome?

   A) 84 B) 90 C) 104 D) 110 E) 130

3. Petra, Quinten, Rakhi, Salome, and Teun organise a badminton tournament consisting of five rounds. In each round, two players play against each other and a third player is the referee. The other two players rest during the round. Everyone plays twice and is the referee once. Nobody plays two matches in a row and the referee of a match always rests in the next round.
Salome and Teun face each other in the first round. In the third round, Rakhi plays against Salome, while Quinten is resting. Who is the referee of the fifth round?

   A) Petra B) Quinten C) Rakhi D) Salome E) Teun

4. The sides of a triangle have lengths 13, x, and $2x$. Here x is an integer.
How many possibilities are there for x?

   A) 2 B) 6 C) 7 D) 8 E) 12

PLEASE CONTINUE ON THE OTHER SIDE

5. On a long street, there are four houses, numbered from 1 to 4, where the distances between the houses are all distinct. The houses have their front door directly on the street. There are eight people living in the first house, two people each in the second and the third house, and three people in the fourth house. A new bus stop is constructed in the street, in such a way that the total distance for the 15 inhabitants of the street to the bus stop is as short as possible.
Which house will be closest to the bus stop?

A) House 1   B) House 2   C) House 3

D) House 4   E) That depends on the distances between the houses.

6. Ayman writes down the numbers 1 through 10 in a sequence in some order, writes down the nine (positive) differences between adjacent numbers and computes the sum of these differences. The result is called the *dynamic* of the sequence. For example, the dynamic of the sequence $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ is 9, and the dynamic of the sequence $2, 1, 3, 10, 4, 5, 9, 6, 8, 7$ is $1 + 2 + 7 + 6 + 1 + 4 + 3 + 2 + 1 = 27$.
What is the greatest dynamic that such a sequence with the numbers 1 through 10 can have?

A) 41   B) 43   C) 45   D) 47   E) 49

7. There are 25 guests at a party, one of which is Medan. Among the other guests, there are 12 that each shook hands with exactly 18 people. The other 12 each shook hands with exactly 6 people. With how many guests did Medan shake hands?

A) 0   B) 6   C) 12   D) 18   E) 24

8. Sil has a lot of cards, which are yellow on one side and blue on the other. Most cards have a number on both sides. If two cards have the same number on the yellow side, then they have the same number on the blue side. There are also cards with a $\times$ on the yellow side and a $+$ on the blue side. Finally, there are cards which have an $=$ sign on both sides. If you put down a correct multiplication with some of the yellow cards and then turn over these cards, then you get a correct addition in blue. Cards with a 2 on the yellow side have a 2 on the blue side, cards with a 3 on the yellow side have a 3 on the blue side, and cards with a 5 on the yellow side have a 5 on the blue side.

All cards are lying on the table with the yellow side facing up. Sil tries to discover what is on the blue side, without turning over the cards. For example, cards with a 6 on the yellow side have a 5 on the blue side, because the yellow expression $2 \times 3 = 6$ must have $2 + 3 = 5$ on the blue back. Cards with a 20 on the yellow side have a 9 on the blue back side, because the yellow expression $2 \times 2 \times 5 = 20$ becomes $2 + 2 + 5 = 9$ in blue. The back of a yellow card containing a fraction, for example $\frac{5}{3}$, can be determined using $\frac{5}{3} \times 3 = 5$, which becomes $2 + 3 = 5$ when flipped; hence on the blue side is a 2.

For which of the following numbers on the yellow side will there be a negative number on the blue side?

A) $\frac{9}{8}$ B) $\frac{25}{27}$ C) $\frac{32}{27}$ D) $\frac{64}{81}$ E) $\frac{128}{125}$