Junior Wiskunde Olympiade
Problems part 2

Saturday 28 September 2019
Vrije Universiteit Amsterdam

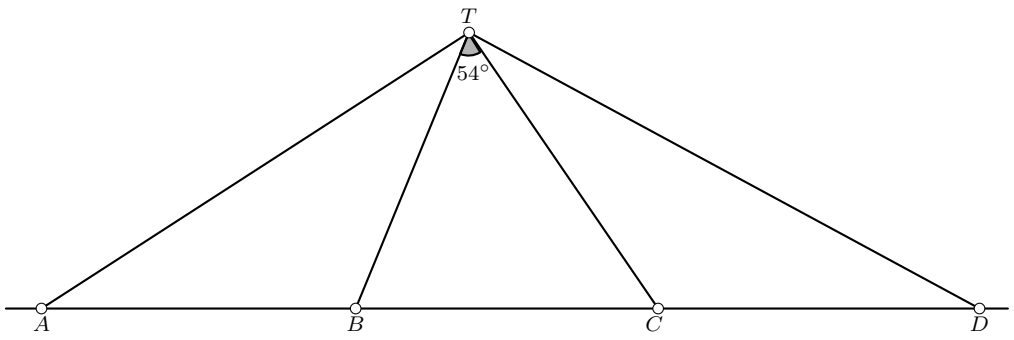
- The problems in part 2 are open questions. Write down your answer on the form at the indicated spot. Calculations or explanations are not necessary.
- Each correct and complete answer is awarded 3 points. For a wrong answer no points are deducted.
- You are allowed to use draft paper. The use of compass, ruler or set square is allowed. Calculators and comparable devices are not allowed.
- You have 45 minutes to solve these problems. **Good luck!**

1. We compute the square of each of the numbers from 1 to 2019. We take the last digit from each of the resulting squares, and then we add those 2019 digits together.
What number do we get?

2. The numbers $abcd$ and $dcba$ consist of the same four digits a, b, c, and d, but in opposite orders. When we add the two numbers, we get 13552.
Determine $a + b + c + d$.

3. One hundred students wear shirts numbered from 1 to 100. The students are arranged in a square of ten rows by ten columns. It turns out that adding the ten shirt numbers of the students in any row or any column always yields the same outcome.
Determine that outcome.

4. The four points A, B, C, and D lie on a common line (in this order). There is a point T not on the line such that $|AB| = |BT|$ and $|CD| = |CT|$. Also, angle T of triangle BTC is 54 degrees. In the figure you can see a sketch of the situation; angles and sizes are not necessarily accurate.
Determine angle T of triangle ATD.



5. Mieke has a stack of 21 cards. Mieke repeats the following operation:

> She takes the top two cards from the stack, changes their order, and then puts them at the bottom of the stack (so the top card becomes the bottom card).

Mieke repeats this operation until the cards are back in their original order.
How many times does Mieke perform the operation?

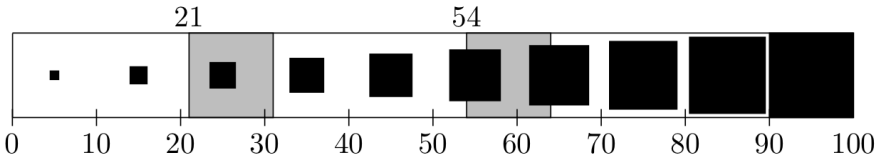
6. The number

$$2222\underbrace{9393\cdots93}_{\text{100 times 93}}919$$

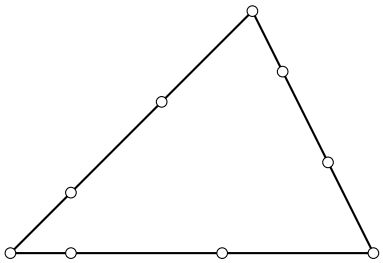
is divided by 2019.
Determine the sum of the digits of the resulting number.

7. On a white strip that is 100 mm long and 10 mm wide, ten black squares are drawn that, from left to right, have sides of length $1, 2, \ldots, 10$ mm. The centre of each black square is in the middle of the strip and $5, 15, \ldots, 95$ mm from the start (left edge) of the strip.

A transparent square is moving along the strip from left to right (indicated in grey). In the figure, two possible positions of the transparent square are depicted: in the first, its left edge is 21 mm from the start of the strip, and in the second, it is 54 mm from the start of the strip. In both cases, less than half of the part underneath the square is coloured black (only 9% in the first case). There is one position in which exactly half of the part of the strip underneath the square is coloured black.
Determine, for that position, how far the left edge of the square is from the start of the strip.



8. We are given a triangle with an additional two points on each side. So in total, there are nine points (see figure).



We want to choose three of the nine points that are not on one line. For example, we could choose (1) the three vertices of the triangle, or (2) the left vertex and the two additional points on the opposite side.
How many possible choices are there in total, including the two examples given?