Junior Wiskunde Olympiade
Problems part 1

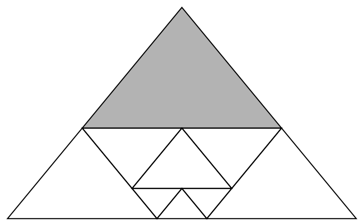
Saturday 28 September 2019
Vrije Universiteit Amsterdam

- The problems in part 1 are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form.
- A correct answer is awarded 2 points, for a wrong answer no points are deducted.
- You are allowed to use draft paper. The use of compass, ruler or set square is allowed. Calculators and comparable devices are not allowed.
- You have 45 minutes to finish these problems. **Good luck!**

1. At a conference, there were participants from four countries: the Netherlands, Belgium, Germany, and France. There were three times as many participants from the Netherlands as there were Belgians, and three times as many Germans as French. Five of the participants counted the total number of participants (including themselves). They counted 366, 367, 368, 369, and 370 participants, respectively. Only one of them got the right answer.
What is the the correct number of participants?

A) 366 B) 367 C) 368 D) 369 E) 370

2. We completely cover a big isosceles triangle with triangles that are similar to the big triangle as in the figure on the right.
What part of the area of the big triangle is covered by the top triangle (indicated in grey)?



A) $\frac{1}{4}$ B) $\frac{2}{7}$ C) $\frac{5}{16}$ D) $\frac{16}{49}$ E) $\frac{1}{3}$

3. In the puzzle below, a, b, c, d, and e are nonzero digits such that the two calculations are correct. The digits need not be distinct.
How many solutions are there for which $a < b$?

$$ab \times ab = cde \qquad \text{and} \qquad ba \times ba = edc.$$

A) 1 B) 2 C) 3 D) 4 E) 5

4. We say that a number is a *child* of another number if we can get it by placing between any two digits of the other number either nothing, a $+$, or a $\times$. For example, 145 and 5 are children of 12121 because $145 = 12 \times 12 + 1$ and $5 = 1 + 2 \times 1 + 2 \times 1$. The number 15 is both a child of 12121 and of 33333, because $12 + 1 + 2 \times 1 = 15 = 3 + 3 + 3 + 3 + 3$.
Which of the following numbers is also a child of both 12121 and 33333?

A) 18 B) 34 C) 39 D) 42 E) 45

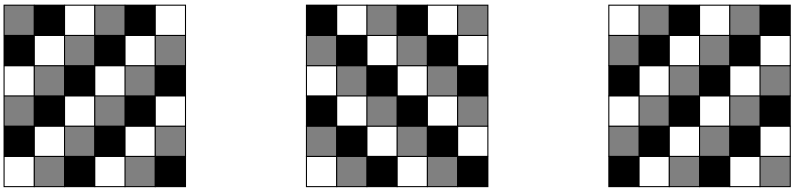
5. In a class, 30 students did a test. Every student got a grade that was an integer from 1 to 10. The grade 8 was given more often than any of the other grades.
What is the smallest possible average grade of the students?

A) $3\frac{8}{15}$ B) $3\frac{2}{3}$ C) $3\frac{5}{6}$ D) $4\frac{11}{30}$ E) $4\frac{8}{15}$

6. We want to colour the 36 squares of a 6×6 board. Every square must be coloured white, grey, or black, and the following requirement must be met:

Three adjacent squares in the same row or column, must always have three different colours.

We say that two colourings are *truly different* if you cannot get one from the other by rotating the board. Below, you can see three colourings that meet the requirement. The first and second colouring are truly different, but the third is the same as the second after rotating.



How many truly different colourings meet the requirement (including the two from the figure)?

A) 2 B) 3 C) 4 D) 6 E) 12

7. Point D lies on side BC of triangle ABC. Angle A in triangle ABD is equal to angle C in triangle ABC, and angle A in triangle ACD is equal to angle B in triangle ABC.

The given information is not enough to derive the exact shape of triangle ABC. However, you can still derive that one of the given statements below is *always false*. Which statement is it? *By $|AB|$ we denote the length of line segment AB.*

A) $|AD| < |AC|$
B) $|AC| < |AB|$
C) $|AB| < |BC|$
D) $|AD| \times |CD| < |AB| \times |AC|$
E) $|AB| \times |AC| < |AD| \times |BC|$

8. Five smart students are sitting in a circle. The teacher gives one or more marbles to each of them. He explains that he has handed out a total of 18 marbles, and that everyone got a different number of marbles. Each student is allowed to see his own number of marbles, as well as the number of marbles of his neighbour on the left and his neighbour on the right.

Using only this information, each student must try to logically deduce the difference between the numbers of marbles of the two students opposite to him. The teacher has distributed the marbles in such a way as to minimise the number of students that are able to do this. How many students can do it?

A) 0 B) 1 C) 2 D) 3 E) 5