Junior Wiskunde Olympiade
Problems part 2

Saturday 30 September 2017
Vrije Universiteit Amsterdam

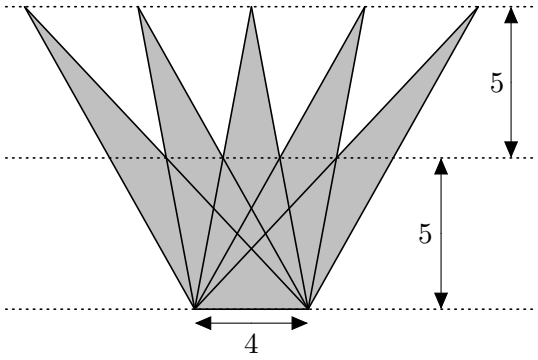
- The problems in part 2 are open questions. Write down your answer on the form at the indicated spot. Calculations or explanations are not necessary.
- Each correct answer is awarded 3 points. For a wrong answer no points are deducted.
- You are allowed to use draft paper. The use of compass, ruler or set square is allowed. Calculators and comparable devices are not allowed.
- You have 45 minutes to solve these problems. **Good luck!**

1. Stef has 18 coins of which 17 are identical, but one is slightly lighter than the other coins. Together, the 18 coins weigh 214 grams. Stef removes two of the coins and weighs the remaining 16 coins. Together the 16 coins weigh 190 grams.
How much does the lighter coin weigh?

2. We say that a positive integer is *balanced* if the average of the first two digits is 2, the average of the first three digits (if they all exist) is 3, the average of the first four digits (if they all exist) is 4, et cetera.
What is the largest balanced number?

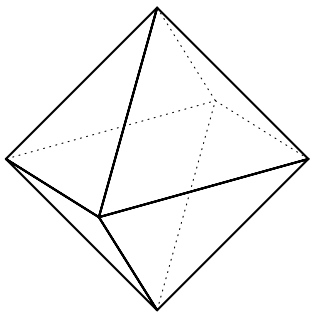
3. What is the area of the crown-shaped area?

Note that the figure is not drawn to scale.



4. A clock has the numbers 1 to 12 for indicating the hours. Ernie has interchanged these twelve numbers in such a way that any two adjacent numbers on the clock differ by either 2 or 3. Fortunately, the number 12 is still in the right place, but the 9 is where the 1 is supposed to be.
What number is in the place where the 9 is supposed to be?

5. The numbers 1 to 8 are assigned to the eight faces of an octahedron. For each vertex, we compute the sum of the four numbers on the faces meeting in that vertex. For four of the vertices we get the same outcome. For a fifth vertex the outcome is 16.
What is the outcome for the sixth vertex?



PLEASE CONTINUE ON THE OTHER SIDE

6. When a, b, c, and d are digits, we denote by $\overline{abcd}$ the number composed of those four digits. The numbers $\overline{abcd}$ and $\overline{cbad}$ are both perfect squares. The number $\overline{bad}$ is the cube of a positive integer.
Determine the number $\overline{abcd}$.

7. An employee at the supermarket is stacking crisps canisters. There are two types of canisters: small ones and large ones. Three small canisters stack to the same height as one large canister. The employee makes a stack of 12 small canisters. Next to it, he makes more stacks of the same height, but all stacks are different. (If one stack starts with a small canister followed by a large one, and the other starts with a large one followed by a small one, the two stacks are different.) How many different stacks can he make, including the first stack?

8. You may choose any number consisting of five different digits. The digit 0 *cannot* be used. The next step is to choose two adjacent digits and switch their positions. You may perform this step five times in total. Finally, you compute the difference between the initially chosen number and the final number obtained after switching.
What is the largest possible difference that can be obtained?

Example. Suppose you choose the number 47632. Then you could switch digits to obtain, in this order, the numbers 46732, 46372, 46327, 43627, and 34627. Then, the difference between the initial number and the final number is $47632 - 34627 = 13005$.