Final round
Dutch Mathematical Olympiad

Version klas 5

Friday 12 September 2025
Technische Universiteit Eindhoven

- Available time: 3 hours.
- Each problem is worth 10 points. Points can also be awarded for partial solutions.
- Write down all the steps of your argumentation. A clear reasoning is just as important as the final answer.
- Formula sheets, calculators, and other electronic devices are not allowed. You can only bring a pen, ruler (set square), compass and your math skills.
- Use a separate sheet for each problem and also hand in your draft sheets (for each problem separately!). Good luck!

1. Behind three of the 25 squares of a scratch card with 5×5 squares a prize is hidden. It is only known that those three squares are all next to each other and together form a 3×1-rectangle (horizontally or vertically). You pay for the scratch card an amount of euros that is equal to the number of squares you are allowed to scratch open (so for 6 euros you are allowed to scratch open 6 squares).
What is the least amount of euros you have to pay for the scratch card to make sure you win a prize?

2. Consider the equation

$$n! - (n-1)n = 4p^2$$

where n is a positive integer and p is a prime number. (Here $n!$ represents the product of the numbers 1 to n. For example, $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$.)
Find all solutions (n, p).

3. Amber has a square consisting of 10×10 square boxes. One at a time, she randomly chooses one of the hundred squares and fills it with a number. The number she puts in a square is always equal to the total number of squares already filled in that square's row and column. So in the first square she chooses she writes a 0, in the second square a 0 or a 1, depending on whether it is in a different row and a different column or not, et cetera. So in each square there will be a number from 0 to 18 inclusive.
After Amber has written a number in each square, she adds up all the numbers. What is/are the possible result(s) of this addition?

4. Within a triangle ABC lies a point P with the property that $\angle ACP = \angle BAP$ and $\angle BCP = \angle ABP$. Prove that P lies on the median of triangle ABC from vertex C. (The median is the line passing through vertex C and the midpoint of side AB.)

5. The vertices of a regular n-gon are numbered clockwise from 0 to $n-1$. We have n pawns, also numbered from 0 to $n-1$, and we put one pawn on each vertex. For each pawn, we consider how many vertices it has to move clockwise to get to the vertex with its own number. We want to place the pawns such that those n numbers are different.

(a) Show that this can always be done if n is odd.
(b) Show that this cannot be done if n is even.