1. Daan distributes the numbers 1 to 9 over the nine squares of a $3 \times 3$-table (each square receives exactly one number). Then, in each row, Daan circles the median number (the number that is neither the smallest nor the largest of the three). For example, if the numbers 8, 1, and 2 are in one row, he circles the number 2. He does the same for each column and each of the two diagonals. If a number is already circled, he does not circle it again.

He calls the result of this process a median table. Above, you can see a median table that has 5 circled numbers.

(a) What is the smallest possible number of circled numbers in a median table?

Prove that a smaller number is not possible and give an example in which a minimum number of numbers is circled.

(b) What is the largest possible number of circled numbers in a median table?

Prove that a larger number is not possible and give an example in which a maximum number of numbers is circled.

2. We consider number sequences $a_1, a_2, a_3, \ldots$ such that $a_{n+1} = \frac{a_n + a_1}{a_n + 1}$ holds for all $n \geq 1$.

(a) Suppose that $a_1 = -3$. Compute $a_{2020}$.

(b) Suppose that $a_1 = 2$. Prove that $\frac{4}{3} \leq a_n \leq \frac{3}{2}$ holds for all $n \geq 2$. 
3. Given is a parallelogram $ABCD$ with $\angle A < 90^\circ$ and $|AB| < |BC|$. The angular bisector of angle $A$ intersects side $BC$ in $M$ and intersects the extension of $DC$ in $N$. Point $O$ is the centre of the circle through $M$, $C$, and $N$.

(a) Prove that triangles $OCM$ and $OCN$ are congruent.
(b) Prove that $\angle OBC = \angle ODC$.

4. Determine all pairs of integers $(x, y)$ for which there exist a prime number $p$ and an integer $a$ such that $x^2 + y^2 = p$ and $(x + y)^2 - a^2 = p$.

5. Sabine has a very large collection of shells. She decides to give part of her collection to her sister. On the first day, she lines up all her shells. She takes the shells that are in a position that is a perfect square (the first, fourth, ninth, sixteenth, etc. shell), and gives them to her sister. On the second day, she lines up her remaining shells. Again, she takes the shells that are in a position that is a perfect square, and gives them to her sister. She repeats this process every day.

The 27th day is the first day that she ends up with fewer than 1000 shells. The 28th day she ends up with a number of shells that is a perfect square for the tenth time. What are the possible numbers of shells that Sabine could have had in the very beginning?