Final Round klas 5 & klas 4 en lager
Dutch Mathematical Olympiad

Friday 18 September 2015
Technische Universiteit Eindhoven

- Available time: 3 hours.
- Each problem is worth 10 points. Points can also be awarded to partial solutions.
- Write down all the steps of your argumentation. A clear reasoning is just as important as the final answer.
- Calculators and formula sheets are not allowed. You can only bring a pen, ruler (set square), compass and your math skills.
- Use a separate sheet for each problem and also hand in your draft sheets (for each problem separately!). Good luck!

1. We make groups of numbers. Each group consists of five distinct numbers. A number may occur in multiple groups. For any two groups, there are exactly four numbers that occur in both groups.

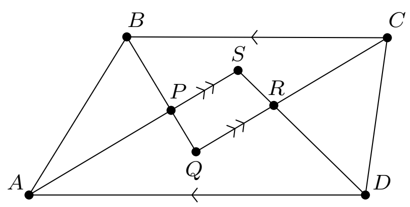
(a) Determine whether it is possible to make 2015 groups.

(b) If all groups together must contain exactly *six* distinct numbers, what is the greatest number of groups that you can make?

(c) If all groups together must contain exactly *seven* distinct numbers, what is the greatest number of groups that you can make?

2. On a 1000×1000-board we put dominoes, in such a way that each domino covers exactly two squares on the board. Moreover, two dominoes are not allowed to be adjacent, but are allowed to touch in a vertex.
Determine the maximum number of dominoes that we can put on the board in this way.
Attention: you have to really prove *that a greater number of dominoes is impossible.*

3. In quadrilateral $ABCD$ sides BC and AD are parallel. In each of the four vertices we draw an angular bisector. The angular bisectors of angles A and B intersect in point P, those of angles B and C intersect in point Q, those of angles C and D intersect in point R, and those of angles D and A intersect in point S. Suppose that PS is parallel to QR.
Prove that $|AB| = |CD|$.
Attention: the figure is not drawn to scale.



4. Find all pairs of prime numbers (p, q) for which

$$7pq^2 + p = q^3 + 43p^3 + 1.$$

5. Given are (not necessarily positive) real numbers a, b, and c for which

$$|a - b| \geqslant |c|, \qquad |b - c| \geqslant |a|, \quad \text{and} \quad |c - a| \geqslant |b|.$$

Here $|x|$ is the absolute value of x, i.e. $|x| = x$ if $x \geqslant 0$ and $|x| = -x$ if $x < 0$.
Prove that one of the numbers a, b, and c is the sum of the other two.