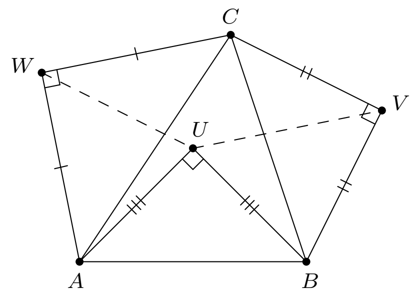
Final round klas 6
Dutch Mathematical Olympiad

Friday 12 September 2014
Technische Universiteit Eindhoven

- Available time: 3 hours.
- Each problem is worth 10 points. Points can also be awarded to partial solutions.
- Write down all the steps of your argumentation. A clear reasoning is just as important as the final answer.
- Calculators and formula sheets are not allowed. You can only bring a pen, ruler (set square), compass and your math skills.
- Use a separate sheet for each problem and also hand in your draft sheets (for each problem separately!). Good luck!

1. Determine all triples (a, b, c), where a, b, and c are positive integers that satisfy $a \leqslant b \leqslant c$ and $abc = 2(a + b + c)$.

2. On the sides of triangle ABC, isosceles right-angled triangles AUB, CVB, and AWC are placed. These three triangles have their right angles at vertices U, V, and W, respectively. Triangle AUB lies completely inside triangle ABC and triangles CVB and AWC lie completely outside ABC. See the figure. Prove that quadrilateral $UVCW$ is a parallelogram.



3. At a volleyball tournament, each team plays exactly once against each other team. Each game has a winning team, which gets 1 point. The losing team gets 0 points. Draws do not occur. In the final ranking, only one team turns out to have the least number of points (so there is no shared last place). Moreover, each team, except for the team having the least number of points, lost exactly one game against a team that got less points in the final ranking.

a) Prove that the number of teams cannot be equal to 6.

b) Show, by providing an example, that the number of teams could be equal to 7.

4. A quadruple (p, a, b, c) of positive integers is called a *Leiden quadruple* if

- p is an odd prime number,
- a, b, and c are distinct and
- $ab + 1$, $bc + 1$ and $ca + 1$ are divisible by p.

a) Prove that for every Leiden quadruple (p, a, b, c) we have $p + 2 \leqslant \frac{a+b+c}{3}$.

b) Determine all numbers p for which a Leiden quadruple (p, a, b, c) exists with $p + 2 = \frac{a+b+c}{3}$.

5. We consider the ways to divide a 1 by 1 square into rectangles (of which the sides are parallel to those of the square). All rectangles must have the same *circumference*, but not necessarily the same shape.

a) Is it possible to divide the square into 20 rectangles, each having a circumference of 2.5?

b) Is it possible to divide the square into 30 rectangles, each having a circumference of 2?