Second round
Dutch Mathematical Olympiad

NEDERLANDSE WISKUNDE OLYMPIADE

Friday, March 14, 2025 (π-day)

- Time available: 2.5 hours.
- The competition consists of five B-problems and two C-problems.
- You are not allowed to use a calculators or other electronic device, nor a formula sheet. You can only use a pen, compass, ruler, set square, and of course your mental skills.
- Good luck!

B-problems

In B-problems, you only have to give the answer (e.g. a number). An explanation is not required. You will get 4 points for a correct answer and 0 points for a wrong or incomplete answer. So work calmly and accurately, as a small miscalculation may result in your answer being wrong.
REMEMBER: give your answers in exact and simplified form such as $\frac{11}{81}$ or $2 + \frac{1}{2}\sqrt{5}$ or $\frac{1}{4}\pi + 1$ or 3^{100}.

B1. Rowdy has a large rectangular sheet of paper whose sides have ratio $2025 : 1$. He cuts the paper into two equally sized rectangular pieces along the line connecting the midpoints of the long sides. Then he discards one of those two pieces, examines which sides of the remaining piece are now the long sides of the rectangle, and cuts that piece again into two equally sized rectangular pieces along the line connecting the midpoints of those long sides. He keeps repeating this process until he has cut a total of 100 times.
What is the ratio between the sides of the paper after 100 cuts?

B2. A positive integer is called *rolling* if every two adjacent digits of this number differ by exactly 1. Examples of rolling numbers are 2101 and 9876.
How many 4-digit rolling numbers (not starting with a 0) are there?

B3. Sam wants to colour each of the numbers 1 to 7 red or blue. For each number k from 1 to 3, if he colours k blue, then he must colour $2k$ and $2k + 1$ both red.
In how many ways can he colour the numbers 1 to 7 this way?

B4. A rectangular jigsaw puzzle consists of corner pieces, edge pieces and centre pieces (all pieces not on a corner or at the edge). The pieces fit neatly into rows and columns. Both the number of pieces in the length and the number of pieces in the width do not exceed 50. The number of corner pieces plus the number of edge pieces is exactly 22% of the total number of pieces.
How many pieces does the whole puzzle consist of?

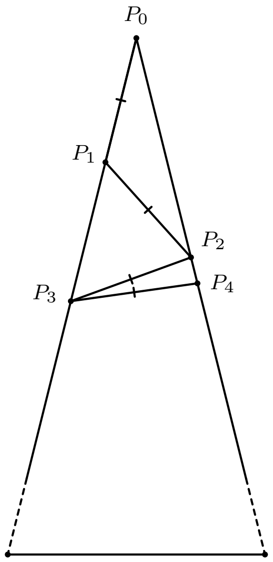
B5. Triangle ABC is an isosceles right-angled triangle with a right angle at C. Point D is the midpoint of BC. The line through D perpendicular to AB intersects the extension of AC in E. The extension of AD intersects BE in G.
Determine the length $|AG|$ of the line segment AG if it is given that the area of triangle ABC equals 100.

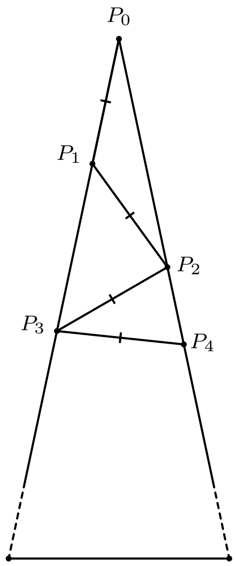
PLEASE CONTINUE ON THE OTHER SIDE

C-problems

For the C-problems not only the answer is important; you also have to write down a clear reasoning that shows the correctness of your answer. A correct and well-explained answer is awarded 10 points. Partial solutions may also be worth some points. Therefore, write everything down clearly and hand in your drafts.
ATTENTION: Use separate sheets of paper for each C-problem and also hand in the drafts for each problem separately.

C1. A monkey sits at the top P_0 of a very tall isosceles triangle with top angle α and a horizontal base. He always makes equally large jumps, landing alternately on the left and right sides of the triangle. He never jumps back to his previous position, but always to a position further from P_0 than the previous position on that side. If that is no longer possible, he stops jumping. The points at which the monkey successively lands are called $P_1, P_2, \ldots, P_n$. Below are two examples for $n = 4$.





(a) Suppose the monkey jumps four times ($n = 4$) and the last jump is horizontal, i.e. $|P_0P_3| = |P_0P_4|$. Calculate the top angle of the triangle.

(b) Suppose the top angle of the triangle is 6 degrees. What is the maximum number of times the monkey can jump? (The triangle is so high that he never reaches the ground.)

C2. We consider the following equation in x and y:

$$(y + x^2 - 1)(y - x^2 + 1) = 4xy.$$

(a) Suppose $x = 10$. What are the possible values of y for which (x, y) is a solution of the equation?

(b) Suppose (x, y) is a solution of the equation where x is an integer. Prove that y or $-y$ is the square of an integer.