Second round
Dutch Mathematical Olympiad

Friday 11 March 2022

- Time available: 2.5 hours.
- The competition consists of five B-problems and two C-problems.
- Formula sheets and calculators are not allowed. You can only use a pen, compass, ruler, set square, and of course your mental skills.
- Good luck!

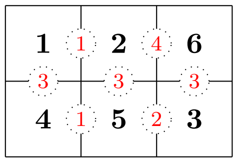
B-problems

For the B-problems you only have to give the answer (for example, a number). No explanation is required. A correct answer is awarded 4 points. For a wrong or incomplete answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.

NOTE: All answers should be given in exact and reduced form, like $\frac{11}{81}$, or $2+\frac{1}{2}\sqrt{5}$, or $\frac{1}{4}\pi+1$, or 3^{100}.

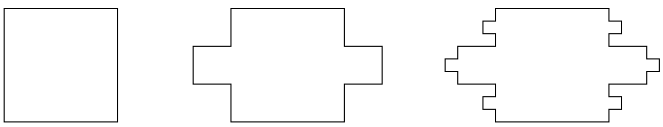
B1. In a 3×2 rectangle, we put the numbers 1 to 6 in the squares in such a way that each number occurs exactly once. The *score* of such a distribution is determined as follows: for each two adjacent squares we compute the difference between their two numbers and we add up all these differences. In the example on the right, the differences are indicated in red. This distribution has score 17.

What is the smallest possible score of such a distribution?



B2. For how many integers n with $1 \leqslant n \leqslant 800$ is the number $8n+1$ a square?

B3. We start with a square with side length 1. During the first minute, small squares with side length $\frac{1}{3}$ grow on the middle of the vertical sides. During the next minute, on the middle of each vertical line segment in the new figure, a new small square grows, whose sides have length $\frac{1}{3}$ of these line segments. Below you can see the situation after 0, 1, and 2 minutes.



This process continues like this. Each minute, on the middle of each vertical line segment a new square grows, whose sides are $\frac{1}{3}$ of the length of that line segment. After one hour this process of new squares growing on the figure has happened 60 times.

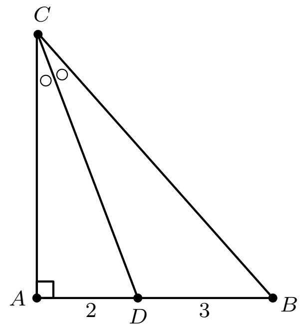
What is the circumference of the figure after one hour?

B4. The candy store sells chocolates in the flavours white, milk, and dark. You can buy them in three types of coloured boxes. The three boxes have the following contents:

- Gold: 2 white, 3 milk, 1 dark,
- Silver: 1 white, 2 milk, 4 dark,
- Bronze: 5 white, 1 milk, 2 dark.

Lavinia buys some boxes of chocolates (at least one) and when she gets home, it turns out she has exactly the same number of chocolates of each flavour.
At least how many boxes did Lavinia buy?

B5. In triangle ABC, angle A is a right angle. A point D lies on line segment AB in such a way that the angles ACD and BCD are equal. Moreover, $|AD| = 2$ and $|BD| = 3$.
What is the length of line segment CD?



C-problems

For the C-problems not only the answer is important; you also have to write down a clear reasoning that shows the correctness of your answer. A correct and well-explained answer is awarded 10 points. Partial solutions may also be worth some points. Therefore, write everything down clearly and hand in your drafts.
ATTENTION: Use separate sheets of paper for each C-problem and also hand in the drafts for each problem separately.

C1. Alicia writes down a distinct integers on a piece of paper and Britt writes down b distinct integers on another piece of paper. Alicia wrote down at least one integer that Britt did not write down, and Britt wrote at least one integer down that Alicia did not write down. Vera counts the number of distinct integers on the two pieces of paper; let this number of distinct integers be v. Daan counts how many of the integers that have been written down by Alicia, have also been written down by Britt; let d be this number. For example, if Alicia wrote down 1, 2, and 5, and Britt wrote down 2, 5, 7, and 8, then we have $a = 3$ and $b = 4$ while $v = 5$ and $d = 2$.

(a) Find an example for which $a = b = 2022$ and $a \cdot b = d \cdot (v + d)$.

(b) Is is possible that $a \cdot b = d \cdot (v + 4)$? Give an example or prove that it is impossible.

(c) Is it possible that $a \cdot b = d \cdot v$? Give an example or prove that it is impossible.

C2. We call a positive integer *sunny* if it has four digits and if moreover each of the two digits on the outside is exactly 1 larger than the digit next to it. The numbers 8723 and 1001 for example are sunny, but 1234 and 87245 are not.

(a) How many sunny numbers are there such that twice the number is again a sunny number?

(b) Prove that every sunny number greater than 2000 is divisible by a three-digit number with a 9 in the middle.