Nederlandse Wiskunde Olympiade voor Bedrijven

Friday, 30 January 2026

Solution uitsmijter

Problem

We consider two points A and B with a mutual distance of $|AB| = 1$. For each real number $\lambda > 1$, there exists a point C on the line segment AB such that $\frac{|AC|}{|BC|} = \lambda$. Moreover, a point D on the extension of the line segment AB exists such that $\frac{|AD|}{|BD|} = \lambda$ as well.

(a) Suppose that $\lambda = \frac{7}{3}$. Calculate the distance $|CD|$.

(b) Suppose that $|CD| = \frac{60}{11}$. Calculate λ.

(c) Let P be the point of tangency of one of the two tangents through A to the circle with diameter CD. Suppose that $|AP| = \frac{13}{12}$. Calculate λ.

Answers

(a) $|CD| = \frac{21}{20}$.

(b) $\lambda = \frac{6}{5}$.

(c) $\lambda = \frac{13}{5}$.

Solution

Without loss of generality, we can assume that $A = (0, 0)$ and $B = (1, 0)$. Furthermore, there exist $0 < c < 1$ and $d > 1$ such that $C = (c, 0)$ and $D = (d, 0)$. It holds that $\frac{|AC|}{|BC|} = \frac{c}{1-c} = \lambda$ and $\frac{|AD|}{|BD|} = \frac{d}{d-1} = \lambda$.

(a) It holds that $\frac{c}{1-c} = \frac{7}{3}$, so $3c = 7(1 - c)$. This gives us $c = \frac{7}{10}$. From $\frac{d}{d-1} = \frac{7}{3}$ it follows that $3d = 7(d - 1)$, so $d = \frac{7}{4}$. Now it follows that $|CD| = d - c = \frac{7}{4} - \frac{7}{10} = \frac{21}{20}$.

(b) From $\frac{c}{1-c} = \lambda = \frac{d}{d-1}$ it follows that $c = \frac{\lambda}{\lambda+1}$ and $d = \frac{\lambda}{\lambda-1}$, so $|CD| = d - c = \frac{\lambda}{\lambda-1} - \frac{\lambda}{\lambda+1} = \frac{\lambda^2+\lambda}{\lambda^2-1} - \frac{\lambda^2-\lambda}{\lambda^2-1} = \frac{2\lambda}{\lambda^2-1}$. With $|CD| = \frac{60}{11}$, this gives us $\frac{2\lambda}{\lambda^2-1} = \frac{60}{11}$, so $11\lambda = 30\lambda^2 - 30$ and hence $30\lambda^2 - 11\lambda - 30 = 0$. The discriminant of this equation is $11^2 - 4 \cdot 30 \cdot -30 = 121 + 3600$. Because $3600 + 121 = 60^2 + 2 \cdot 60 + 1 = (60 + 1)^2$, the square root of the discriminant is 61. Now we find that $\lambda = \frac{11 \pm 61}{60}$. Because $\lambda > 1$, we must have $\lambda = \frac{72}{60} = \frac{6}{5}$.

(c) Let M be the center of the circle with diameter CD. Since AP is a tangent of the circle, AP is perpendicular to the radius PM of the circle. It follows from the Pythagorean theorem that $|AP|^2 + |PM|^2 = |AM|^2$, so $|AP|^2 = |AM|^2 - |PM|^2$. Hence $|AP|^2 = (|AM| + |PM|) \cdot (|AM| - |PM|) = |AD| \cdot |AC| = d \cdot c$ because $|PM| = |DM| = |CM|$.

Previously we calculated $c = \frac{\lambda}{\lambda+1}$ and $d = \frac{\lambda}{\lambda-1}$, so $|AP|^2 = c \cdot d = \frac{\lambda^2}{\lambda^2-1}$. Because $|AP| = \frac{13}{12}$, we have $\frac{\lambda^2}{\lambda^2-1} = \frac{169}{144}$, so $144\lambda^2 = 169\lambda^2 - 169$. It follows that $\lambda^2 = \frac{169}{25}$, so $\lambda = \frac{13}{5}$.