

# First round

## Dutch Mathematical Olympiad



20 January – 31 January 2025

- Time available: 2 hours (120 minutes).
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- Each B-problem requires a short answer (e.g. a number) without further explanation. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.  
NOTE: All answers should be given in exact and simplified form, like  $\frac{11}{81}$ ,  $2 + \frac{1}{2}\sqrt{5}$ ,  $\frac{1}{4}\pi + 1$ , or  $3^{100}$ .
- Formula sheets, calculators, and other electronic devices are not allowed. You can only use a pen, paper, compass, ruler or set square and of course your mental skills.
- After the contest, hand in your answer sheet, this problem sheet and any scrap paper. The problems and solutions will be available from 1 February on the website: [www.wiskundeolympiade.nl](http://www.wiskundeolympiade.nl).
- Good luck!

### A-problems

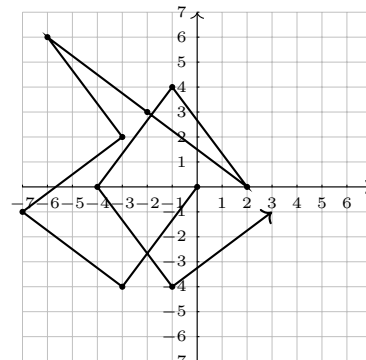
1. Five gnomes each have a red or blue hat on. The gnomes know that there are a total of three red hats and two blue hats. Gnome A looks at the hats of gnomes C and D and cannot deduce which colour hat he is wearing. Gnome B looks at the hats of gnomes C and E and also cannot deduce which colour hat he is wearing. Finally, gnome C looks at the hats of gnomes D and E and also cannot deduce which colour hat he is wearing.  
Which of the following statements is definitely true?  
A) At least one of gnomes A and B is wearing a blue hat.  
B) At least one of gnomes A and B is wearing a red hat.  
C) Gnome C is wearing a red hat.  
D) At least one of gnomes D and E is wearing a blue hat.  
E) Gnomes D and E are both wearing a red hat.
2. The three sisters Alexa, Bente, and Charissa each have an age (in years) that consists of one digit, where Alexa is the youngest and Charissa the oldest. If you put Bente's age directly behind Alexa's, so that a two-digit number is formed, you get exactly the age of their father. Charissa's age behind Alexa's gives exactly the age of their mother. Charissa's age behind Bente's age gives their grandfather's age. Grandfather is 32 years older than father. If you add up the ages of father and mother, you get grandfather's age plus Bente's age.  
How old is Alexa?  
A) 2      B) 3      C) 4      D) 5      E) 6
3. Laura is baking muffins and chocolate cake. In doing so, she uses exactly the same amount of sugar as flour. In the muffins, she puts twice as much flour as sugar. In the chocolate cake, she puts  $\frac{5}{4}$  times as much sugar as flour.  
What proportion of flour does Laura use for the muffins?  
A)  $\frac{1}{6}$       B)  $\frac{1}{3}$       C)  $\frac{1}{2}$       D)  $\frac{2}{3}$       E)  $\frac{5}{6}$

PLEASE CONTINUE ON THE OTHER SIDE

4. Of which of the following numbers is  $2021 \cdot 2022 \cdot 2023 \cdot 2024 + 1$  the square?

- A)  $2021 \cdot 2023 - 1$       B)  $2021 \cdot 2024 - 1$       C)  $2022 \cdot 2024 - 1$   
 D)  $2022 \cdot 2023 - 1$       E)  $2023 \cdot 2024 - 1$

5. On a grid consisting of square boxes, a path from the origin is drawn by making one move at a time. A move consists of moving three squares horizontally while moving four vertically, or four horizontally while moving three vertically. The figure shows such a path, which consists of ten moves.



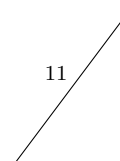
What is the minimum number of moves you need in order to reach the point  $(1, 0)$  from the origin?

- A) 5      B) 7      C) 9      D) 11      E) 13

6. We consider a triangle with an angle of 90 degrees. The hypotenuse has length 11 and the perimeter of the triangle is 25.

What is the area of the triangle?

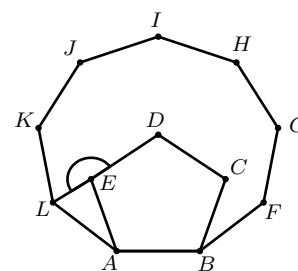
- A) 18      B)  $18\frac{1}{4}$       C)  $18\frac{1}{2}$       D)  $18\frac{3}{4}$       E) 19



7. Given are a regular pentagon  $ABCDE$  and a regular nonagon  $ABFGHIJKL$  as in the figure. Attention: the figure is not drawn to scale.

What is the angle at  $E$  between the legs  $EL$  and  $ED$ ?

- A)  $176^\circ$       B)  $177^\circ$       C)  $178^\circ$       D)  $179^\circ$       E)  $180^\circ$



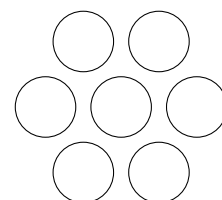
8. Babette puts a cross on the number line at exactly 1001 of the numbers 1 to 2025. Then for each pair of (not necessarily adjacent) crosses, she writes down the distance between them. Then she adds up all these distances.

In how many ways can she put her crosses to get the largest possible outcome?

- A) 1      B) 2      C) 1001      D) 1025      E) 2025

## B-problems

1. Roel wants to surprise his sick friend Alan with a beautiful bouquet of three red and four white roses. He places a rose in the centre and the other six roses around it, as in the picture on the right. There are several ways to arrange these roses in a beautiful bouquet. We consider two ways the same if Roel can rotate the bouquet and get the other bouquet.



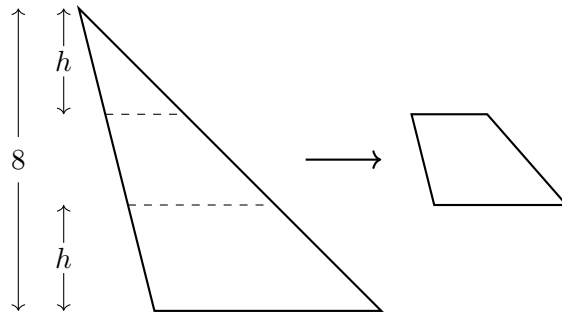
How many ways are there for Roel to arrange his bouquet?

2. On a digital clock, the possible times run from  $00 : 00 : 00$  to  $23 : 59 : 59$ . It is possible to construct five times that together use each of the digits 0 to 9 exactly three times.

In doing so, what is the largest possible time difference between the earliest and latest time? The time difference between, say,  $08 : 34 : 16$  and  $23 : 14 : 27$  is  $14 : 40 : 11$ .

3. From a triangle of height 8, parallel to the base, a piece of height  $h$  is cut away at both the top and bottom (see the illustration as an example). The area of the remaining piece has only one fifth of the area of the original triangle.

What is the height  $h$  of the pieces that are cut off? *Please note that the heights of the cut lines are not to scale.*



4. Lydia writes some one-digit numbers on the chalkboard and wants to add them all up. She does this as follows: first she adds the first two numbers, then she adds the result to the third number, and so on, until she has added all the numbers. After each addition, Lydia writes the (intermediate) answer on the board. For example, if Lydia starts with 3, 9, 5, then at the end the numbers 3, 9, 5, 12, 17 are written on the board (because  $12 = 3 + 9$  and  $17 = 12 + 5$ ): those are the digits 1, 2, 3, 5, 7, and 9. Lydia wants all digits 1 through 9 to appear on the chalkboard at least once when she is done. Moreover, she wants to start with as few numbers as possible. Give an as short as possible sequence of numbers that Lydia could start with.