First round
Dutch Mathematical Olympiad

22 January – 2 February 2024

- Time available: 2 hours (120 minutes).
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- Each B-problem requires a short answer (e.g. a number) without further explanation. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
  NOTE: All answers should be given in exact and simplified form, like $\frac{11}{81}$, $2 + \frac{1}{2}\sqrt{5}$, $\frac{1}{4}\pi + 1$, or 3^{100}.
- Formula sheets, calculators, and other electronic devices are not allowed. You can only use a pen, paper, compass, ruler or set square and of course your mental skills.
- After the contest, hand in your answer sheet, this problem sheet and any scrap paper. The problems and solutions will be available from 3 February on the website: `www.wiskundeolympiade.nl`.
- Good luck!

A-problems

1. We call a positive integer *quadraticular* if every two adjacent digits form a square number. For example, the number 364 is quadraticular because both 36 and 64 are squares.
How many digits does the largest quadraticular number have?

A) 2 B) 3 C) 4 D) 5 E) 6

2. Caitlin took a long walk each day in October. Only on the 16 days when rain was expected, she took an umbrella with her. Of the 31 days of October, the rain forecast was correct on exactly 21 days. Fortunately, on the days when it rained, Caitlin always had her umbrella with her.
On how many days did it not rain?

A) 6 B) 10 C) 16 D) 21 E) 25

3. Montasser makes a sequence of numbers. The first two numbers are 6 and 15. He always makes the next number in the sequence by dividing the last number by its predecessor and multiplying the result by 2. Thus, the third number in the sequence is $\frac{15}{6} \cdot 2 = 5$ and the fourth number is $\frac{5}{15} \cdot 2 = \frac{2}{3}$.
What is the one hundredth number in the sequence?

A) 15 B) 5 C) $\frac{2}{3}$ D) $\frac{4}{15}$ E) $\frac{4}{5}$

4. We place the digits 1 through 9 one by one in a 3×3 grid. The digit 1 may be placed in an arbitrary chosen box; each subsequent digit comes in a box that is horizontally or vertically adjacent to the box that contains the previous digit. See, for example, the picture on the right. We call such a grid a *snake grid*. The score of a box in a snake grid is the sum of the digits in all boxes with one side adjacent to the box. The total score of a snake grid is the sum of the scores of all its boxes. For example, the snake grid in the example (row by row, adding up from left to right) has total score $6+13+10+9+20+19+10+15+14 = 116$. How many possible total scores can such a snake grid have?

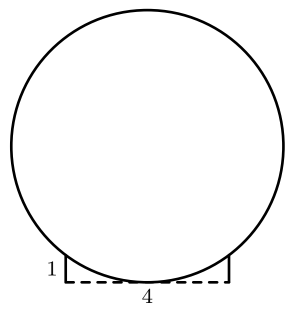
1	2	9
4	3	8
5	6	7

A) 1 B) 2 C) 3 D) 4 E) 5

5. Xander draws five points and a number of infinitely long lines on an infinite sheet of paper. He does this in such a way that on each line there are at least two of those points and that the lines intersect only at points that Xander has drawn.
What is the maximum number of lines Xander could have drawn?

A) 3 B) 4 C) 5 D) 6 E) 7

6. At Matthijs's table tennis club, one keeps the ping-pong balls on a table with cylindrical ball holders. Here is the side view of a ping-pong ball on top of a ball holder. The underside of the ball exactly touches the table. It is known that the ball holder is 4 centimetres wide and 1 centimetre high. How many centimetres is the radius of the ball? *Please note that the picture is not to scale.*



A) $2\frac{1}{3}$ B) $2\frac{1}{2}$ C) $2\frac{2}{3}$ D) $2\frac{5}{6}$ E) 3

7. Exactly one of the following statements is true: which one? *Please note that the numbers a and b need not be integers.*

A) There do not exist $a > 0$ and $b > 0$ with $a \cdot b < \frac{a}{b} < a + b$.
B) There do not exist $a > 0$ and $b > 0$ with $a \cdot b < a + b < \frac{a}{b}$.
C) There do not exist $a > 0$ and $b > 0$ with $a + b < a \cdot b < \frac{a}{b}$.
D) There do not exist $a > 0$ and $b > 0$ with $\frac{a}{b} < a \cdot b < a + b$.
E) A) through D) are false.

8. Birgit has ten candles. Each candle has a burning time of three hours. At one point, she lights one of those candles. From that moment on, she wants the ten candles to burn under the following conditions:

1. each subsequent candle is lighted only after a whole number of hours;
2. at least one candle should burn at all times;
3. no candle is blown out;
4. if in a period between two whole hours one particular candle or combination of candles burns, then it is not allowed that in the next period between two whole hours the same candle or combination of candles burns.

Under those conditions, what is the maximum number of hours until all 10 candles are burnt out?

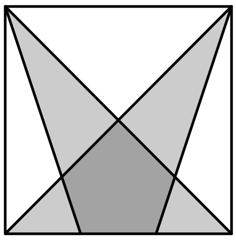
A) 12 B) 15 C) 16 D) 17 E) 18

B-problems

1. Tomorrow, the Janssen family will be travelling by car and they have a nice route in mind. The youngest of the family notes that their planned stopover in Germany is exactly halfway along the route in terms of distance. Father responds: "When we cross the border after 150 kilometres tomorrow, our stopover will only be on one fifth of the remaining route."
How many kilometres long is the Janssen family's route?

2. Floor's class consists of 16 students, including Floor. All students took a test with four questions. Every question was worth a (positive) integer number of points. Each question was marked completely right or completely wrong; no partial points were given. The question that was worth the most points was worth exactly 4 points more than the question worth the least points. All students achieved a different score; Floor herself got everything right.
At least how many points did Floor score?

3. In the figure on the right you see a square with side 1. From the two upper corners of that square, two spotlights shine on the bottom side. The left spotlight shines on exactly two thirds of the bottom side on the right and the right spotlight shines on exactly two thirds of the bottom side on the left. In the figure, you can see that there is a section (in dark grey) that is illuminated by both spotlights.
What is the area of the dark grey region?



4. Pjotr has a bath with a stopper in the bottom that he can fill with two identical taps. The water flows out of both taps at the same constant speed, and the bath empties (if the stopper is not there) at a constant speed.
On Monday, Pjotr fills the bathtub to the brim by turning one tap fully open, then pulls out the stopper and waits for the bathtub to empty again. Only when the bath is empty does he turn the tap off again. On Tuesday, he fills the bath to the brim by turning both taps fully open, after which he pulls out the stopper and waits for the bath to empty again. Only when the bath is empty does he turn the taps off again. On both days, Pjotr spent exactly 45 minutes from opening the tap/taps to closing them again.
How many minutes does it take to completely empty a full bath when both taps are off?