

First round

Dutch Mathematical Olympiad



17–27 January 2022

- Time available: 2 hours (120 minutes).
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- Each B-problem requires a short answer (e.g. a number) without further explanation. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
NOTE: All answers should be given in exact and simplified form, like $\frac{11}{81}$, $2 + \frac{1}{2}\sqrt{5}$, $\frac{1}{4}\pi + 1$, or 3^{100} .
- Formula sheets and calculators are not allowed. You can only use a pen, paper, compass, ruler or set square and of course your mental skills.
- After the contest, hand in your answer sheet, this problem sheet and any scrap paper. The problems and solutions will be available from 29 January on the website: www.wiskundeolympiade.nl.
- Good luck!

A-problems

1. A group of islands consists of a large, a medium and a small island. The total area of the three islands together is 23 km^2 . The difference between the areas of the large and the medium island turns out to be exactly 1 km^2 more than the area of the small island.

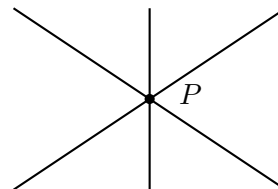
How many km^2 is the area of the large island?

- A) 10 B) 11 C) 12 D) 13 E) 14

2. Kevin draws a point P on a large piece of paper. Then he draws, one by one, straight lines through P .

How many lines does Kevin have to draw at least to make sure that on the piece of paper there are two lines that make an angle of less than 13 degrees?

- A) 9 B) 13 C) 14 D) 27 E) 28



3. Sofie and her grandmother both have their birthday on 1 January. The age of grandma is in six consecutive years an integer multiple of the age of her granddaughter Sofie. This is not true for the seventh year. A few years later the age of grandma is again an integer multiple of the age of Sofie.

How old can grandma be by then?

- A) 63 B) 66 C) 70 D) 90 E) 91

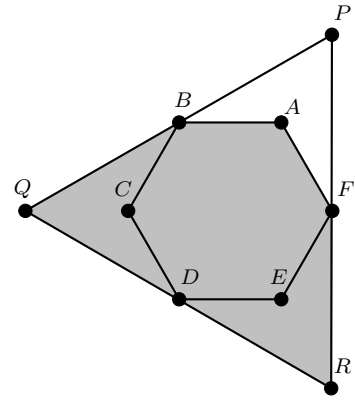
4. When you add the digits of the number 2022, you get 6.

How many 4-digit numbers are there (including 2022) such that, when you add the digits, you get 6? The numbers can not start with the digit 0.

- A) 40 B) 45 C) 50 D) 55 E) 56

PLEASE CONTINUE ON THE OTHER SIDE

5. Consider the equilateral triangle PQR . Inside this triangle the regular hexagon $ABCDEF$ is drawn. Points B , D and F are the midpoints of the sides of the triangle PQR . The area of the pentagon $QBAFR$ is equal to 1. What is the area of the triangle PQR ?



- A) $\frac{11}{10}$ B) $\frac{7}{6}$ C) $\frac{6}{5}$ D) $\frac{5}{4}$ E) $\frac{4}{3}$

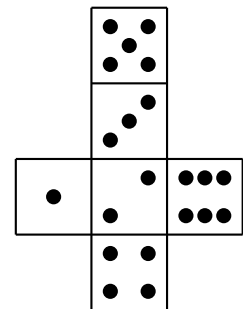
6. A box contains red, white and blue balls. The number of red balls is an even number and the total number of balls in the box is less than 100. The number of white and blue balls together is 4 times as much as the number of red balls. The number of red and blue balls together is 6 times as much as the number of white balls. How many balls are in the box?

- A) 28 B) 30 C) 35 D) 70 E) 84

7. In a tournament with the four teams A, B, C and D, every team played against every other team in three rounds of two simultaneous games. No team won or lost all their games and no game ended in a draw. We know that team A won in the first and third round. Also, team C won in the first round and team D lost in the second round. Five people make a statement about the tournament, but only one of them is telling the truth. Which statement is true?

- A) A and B played against each other in round 1 B) C won from B
 C) A and D played against each other in round 3 D) D won from A
 E) B and C played against each other in round 2

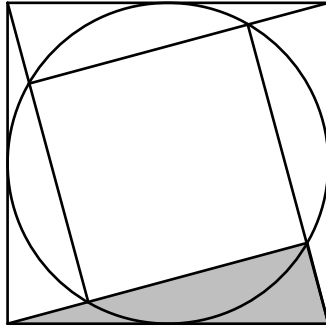
8. Michael prints the net in the figure twice on cardboard and makes it into two identical dices, such that the pips are visible on the outside of the dice. He puts one dice on top of the other to make a small tower. The front face of the lower dice shows 3 pips. The total number of pips on the two faces touching in the middle is equal to 9. The total number of pips on the back of the small tower is three times the total number of pips on the right side of the small tower. How many pips are on the face that touches the ground?



- A) 1 B) 2 C) 4 D) 5 E) 6

B-problems

1. Line up the numbers 1 to 15 such that if you add any two numbers that are next to each other, you get a square number.
What do you get if you add the first and last number from the line?
2. In the figure below the large square has sides of length 6. The circle is tangent to all sides of the large square. The four triangles are exactly the same right angled triangles and are directly next to each other; the small square they enclose has its vertices exactly on the circle.



What is the area of the grey triangle?

3. At a congress all attendees are either a mathematician or a biologist and there is no-one that is both. The mathematicians all know each other and each of them knows four of the biologists. The biologists also all know each other and each of them knows nine of the mathematicians. It turns out that every mathematician knows twice as many people as every biologist. (If person A knows person B, then person B also knows person A.)
How many mathematicians are at the congress?
4. On an 8×8 -board there is a beetle on every square. On a certain moment the division of the beetles on the board changes: every beetle crawls either one square to the left or one square diagonally to the bottom right. If a beetle can make neither of the two movements without falling off the board, it stays on its square.
How many squares can maximally get unoccupied by this change?