First round
Dutch Mathematical Olympiad

18 January – 4 February 2021

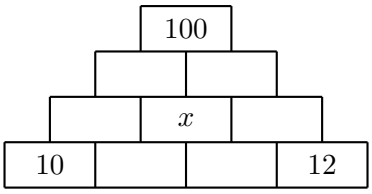
- Time available: 2 hours (120 minutes).
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- Each B-problem requires a short answer (e.g. a number) without further explanation. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
  NOTE: All answers should be given in exact and simplified form, like $\frac{11}{81}$, $2+\frac{1}{2}\sqrt{5}$, $\frac{1}{4}\pi+1$, or 3^{100}.
- Formula sheets and calculators are not allowed. You can only use a pen, paper, compass, ruler or set square and of course your mental skills.
- After the contest, hand in your answer sheet, this problem sheet and any scrap paper. The problems and solutions will be available from 8 February on the website: `www.wiskundeolympiade.nl`.
- Good luck!

A-problems

1. For the integers a, b, c, and d the difference between a and b equals 2, the difference between b and c equals 3, and the difference between c and d equals 4. Which of the following values cannot be the difference between a and d?

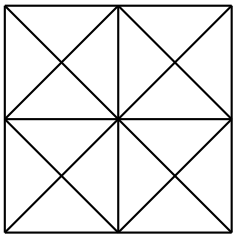
A) 1 B) 3 C) 5 D) 7 E) 9

2. In each square of the top three rows in the pyramid on the right, the number written in that square equals the sum of the numbers in the two squares below it. For three of the squares, the numbers written in them are given.
What number must be written in the square with the x in it?



A) 17 B) 20 C) 23 D) 26 E) 39

3. How many triangles are there in the figure on the right?

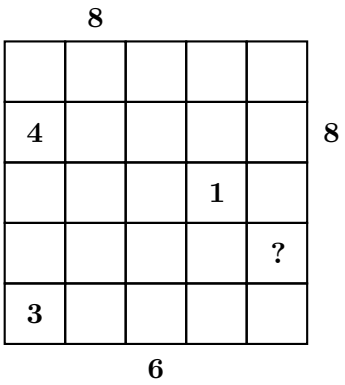


A) 32 B) 36 C) 40 D) 44 E) 64

PLEASE CONTINUE ON THE OTHER SIDE

4. In each square of the field on the right, there is a high-rise building of height 1, 2, 3, 4, or 5, such that the following conditions are satisfied.

- In each (horizontal) row or (vertical) column, each height occurs exactly once.
- The numbers on the side of the square are the sums of the heights of the visible buildings. This concerns the buildings in this particular row or column that are (partially) visible in the side view from the number on the side. For example, if the heights 1, 3, 2, 5, and 4 occur in this order in a row, then the buildings of heights 1, 3, and 5 are visible from the left side, and the buildings of heights 4 and 5 are visible from the right side.



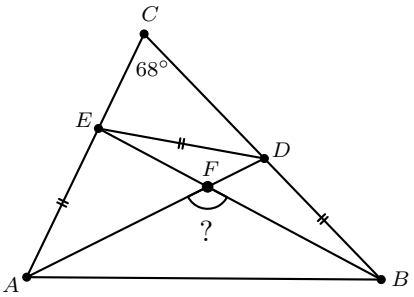
What is the height of the building on the square with the question mark?

A) 1 high B) 2 high C) 3 high D) 4 high E) 5 high

5. The number 1 is written on the blackboard. A *turn* consists of wiping out the number on the board and replacing it by the double of the number, or by the number one smaller. For example, we can replace 1 by 2 (the double) or 0 (one smaller), and if 5 is on the board, we can replace it by 10 or 4.
What is the minimum number of turns needed in order to write the number 2021 on the board?

A) 14 B) 15 C) 16 D) 17 E) 18

6. In triangle ABC, a point D lies on side BC and a point E lies on side AC such that the line segments BD, DE, and AE have the same length. The point F is the intersection between the line segments AD and BE. Angle C is 68°.
What is the size of angle F in triangle AFB?



A) 120° B) 121° C) 122° D) 123° E) 124°

7. The integers 1 to n are written on the board. One of the numbers is wiped out. The average of the remaining numbers is $11\frac{1}{4}$.
Which number has been wiped out?

A) 6 B) 7 C) 11 D) 12 E) 21

8. We order the positive odd integers as follows:

	column 1	column 2	column 3	column 4	column 5	column 6	⋮
row 1	1	3	11	13	29	31	...
row 2	5	9	15	27	33	...	
row 3	7	17	25	35	...		
row 4	19	23	37	...			
row 5	21	39	...				
row 6	41	...					
...	...						

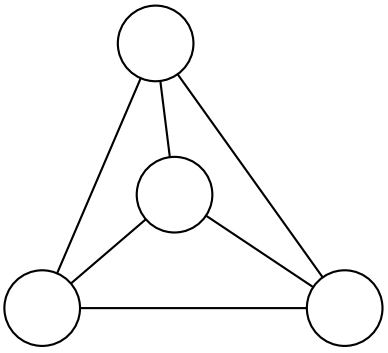
For each odd number we can determine in which row and column it is placed. For example, the number 35 is placed in row 3 and column 4.
What number is placed in row 22 and column 24?

A) 2021 B) 2023 C) 2025 D) 2027 E) 2029

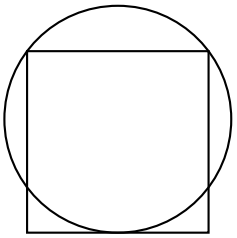
B-problems

1. We have two integers consisting of two digits, and both numbers do not start with a 0. If you add these numbers, you get the number S. If you interchange the two digits of both numbers and add the new numbers, you get $4S$.
Determine all possible pairs of two-digit numbers satisfying these constraints. Make sure to clearly indicate in your answer which numbers form a pair.

2. In the diagram on the right we write a number in each circle. The numbers do not have to be integers or be positive. Next to each line segment, we write the sum of the two numbers in the circles on the end of the line segment. There are two quadruples of numbers that we can write in the circles, such that the numbers next to the line segments are exactly the numbers 0, 1, 2, 3, 4, and 5. For both of these quadruples, we multiply the four numbers in the circles with each other.
Which two results can we get from this multiplication?



3. A circle of radius 1 and a square are given, such that the circle is tangent to one side of the square and also two of the vertices of the square lie on the circle.
What is the length of a side of the square?



4. We consider security codes consisting of four digits. We say that one code *dominates* another code if each digit of the first code is at least as large as the corresponding digit in the second code. For example, 4961 dominates 0761, because $4 \geqslant 0$, $9 \geqslant 7$, $6 \geqslant 6$, and $1 \geqslant 1$. We would like to assign a colour to each security code from 0000 to 9999, but if one code dominates another code then the codes cannot have the same colour.
What is the minimum number of colours that we need in order to do this?