

First round

Dutch Mathematical Olympiad

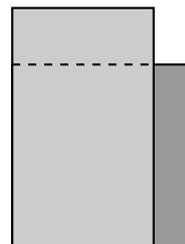


20 January – 30 January 2020

- Time available: 2 hours (120 minutes).
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- Each B-problem requires a short answer (e.g. a number) without further explanation. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
NOTE: All answers should be given in exact and simplified form, like $\frac{11}{81}$, $2 + \frac{1}{2}\sqrt{5}$, $\frac{1}{4}\pi + 1$, or 3^{100} .
- Formula sheets and calculators are not allowed. You can only use a pen, paper, compass, ruler or set square and of course your mental skills.
- After the contest, hand in your answer sheet, this problem sheet and any scrap paper. The problems and solutions will be available from 1 February on the website: www.wiskundeolympiade.nl.
- Good luck!

A-problems

1. Francisca has a square piece of paper whose sides have length 10 cm. She also has a rectangular piece of paper having the exact same area as the square piece of paper. She puts the rectangle right on top of the square, putting the left bottom corner of both pieces of paper in the same spot. Exactly one quarter of the square remains uncovered by the rectangle. What is the length in centimetres of the long side of the rectangle?



- A) 12 B) $12\frac{1}{4}$ C) $12\frac{1}{2}$ D) $12\frac{3}{4}$ E) $13\frac{1}{3}$

2. Each of Kwik, Kwek, and Kwak is lying on two consecutive days of the week and is telling the truth on the other five days. No two of them are lying on the same day. Uncle Donald wants to know who of his nephews ate his sweets. The three nephews know all too well who did it. On Sunday, Kwik says that Kwek ate the sweets. On Monday, Kwik says that it actually was not Kwek who ate the sweets, while Kwak claims that Kwik is innocent. On Tuesday, however, Kwak says that it was Kwik who ate the sweets. Who ate the sweets?

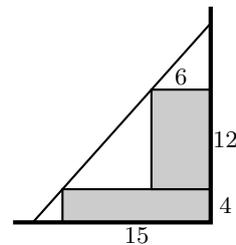
- A) It was Kwik.
B) It was Kwek.
C) It was Kwak.
D) It was either Kwik or Kwek, but you cannot determine who of the two.
E) It was either Kwik or Kwak, but you cannot determine who of the two.

3. We consider numbers with two digits (the first digit cannot be 0). Such a number is called *vain* if the sum of the two digits is greater than or equal to the product of the two digits. For example, the number 36 is *not* vain, as $3 + 6$ is smaller than $3 \cdot 6$. How many numbers with two digits are vain?

- A) 17 B) 18 C) 26 D) 27 E) 37

PLEASE CONTINUE ON THE OTHER SIDE

4. A box measuring 4 dm by 15 dm is shoved against the wall. On top of it, a second box, measuring 12 dm by 6 dm, is placed. A ladder exactly touches the ground, the two boxes and the wall. See the figure (which is not drawn to scale).



What is the length of the ladder in dm?

- A) 30 B) $8\sqrt{15}$ C) 31 D) $22\sqrt{2}$ E) $18\sqrt{3}$

5. On a 4×4 board, there are 16 grasshoppers, each on its own square. At a certain time, each grasshopper jumps to an adjacent square: to the square above, below, left, or right of its current square, but not diagonally and not leaving the board.

What is the maximum number of squares that can be empty after the grasshoppers have jumped?

- A) 8 B) 9 C) 10 D) 11 E) 12

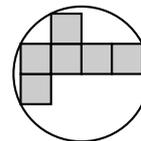
6. In the table below each of the three rows is a correct calculation (the symbol \div denotes division). Also each of the three columns (read from top to bottom) is a correct calculation. However, the digits in the table have been replaced by letters. Different letters represent different digits and no digits are 0.

$$\begin{array}{rclcl} ABC & - & ADF & = & F \\ + & & - & & - \\ ADD & \div & GC & = & C \\ = & & = & & = \\ CEF & \div & GD & = & D \end{array}$$

Which digit does E represent?

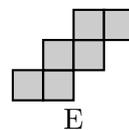
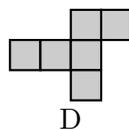
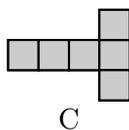
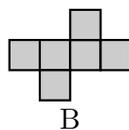
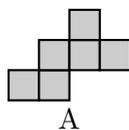
- A) 1 B) 3 C) 5 D) 7 E) 9

7. We consider figures consisting of six squares whose sides have length 1. The *radius* of such a figure is the radius of the smallest circle containing the whole figure. On the right, there is an example of a figure with radius $\sqrt{5}$.

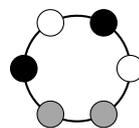
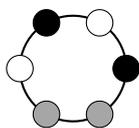
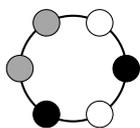


Which of the following five figures has the smallest radius?

- A) A B) B C) C D) D E) E



8. Lieneke is making bracelets with beads. Each bracelet has six beads: two white, two grey, and two black beads. Some bracelets look different on first sight, but are actually not different: by turning or flipping the first one over, it looks the same as the other one. For example, the following three bracelets are the same.



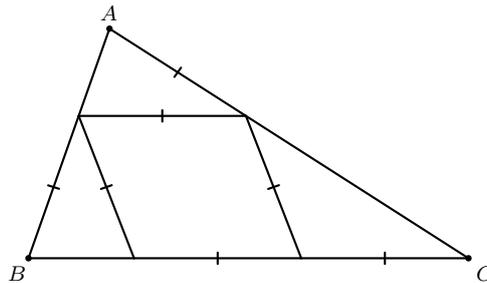
How many really different bracelets can Lieneke make?

- A) 10 B) 11 C) 12 D) 14 E) 15

B-problems

1. By replacing each $*$ in the expression $1 * 2 * 3 * 4 * 5 * \dots * 2019 * 2020$ by a $+$ or a $-$ sign, we get a long calculation. Put the $+$ and $-$ signs in such a way that the outcome is a positive number (greater than 0) which is as small as possible.
What is this outcome?

2. Triangle ABC is subdivided into three isosceles triangles and a rhombus. *Note: the figure is not drawn to scale.*



What is the size of angle C in degrees?

3. Annemiek and Bart each have a note on which they have written three different positive integers. It appears that there is exactly one number that is on both their notes. Moreover, if you add any two different numbers from Annemiek's note, you get one of the numbers on Bart's note. One of the numbers on Annemiek's note is her favourite number, and if you multiply it by 3, you get one of the numbers on Bart's note. Bart's note contains the number 25, his favourite number.
What is Annemiek's favourite number?
4. We consider rows of 2020 coins. Each coin is of denomination 1, 2, or 3. Between two coins of denomination 1, there is at least one other coin. Between two coins of denomination 2, there are at least two other coins. Between two coins of denomination 3, there are at least three other coins.
How many different rows of 2020 coins satisfy these conditions?