First round
Dutch Mathematical Olympiad

23 January – 2 February 2017

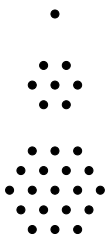
- Time available: 2 hours.
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- Each B-problem requires a short answer (e.g. a number) without further explanation. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
  NOTE: All answers should be given in exact form, like $\frac{11}{81}$, $2+\frac{1}{2}\sqrt{5}$ or $\frac{1}{4}\pi+1$.
- Formula sheets and calculators are not allowed. You can only use a pen, paper, compass, ruler or set square and of course your mental skills.
- After the contest, hand in your answer sheet, this problem sheet and any scrap paper. The problems and solutions will be available from 3 February on the website: `www.wiskundeolympiade.nl`.
- Good luck!

A-problems

1. In a certain year, August has only 4 Mondays and 4 Fridays.
Which day of the week was 31 August that year?

A) Tuesday B) Wednesday C) Thursday
D) Saturday E) Sunday

2. We consider dotted hexagons with $1, 2, 3, \ldots$ dots on each side, see also the picture. The number of dots in such a hexagon is called a *hexagonal number*. The first hexagonal number is 1, the second is 7, and the third is 19.
Which of the following numbers is also a hexagonal number?



A) 81 B) 128 C) 144 D) 169 E) 187

3. Five suspects are arrested in a criminal investigation. Each of them makes one statement:

Eva: "We are all innocent."

Fatima: "Exactly one of us is innocent."

Kees: "Exactly one of us is guilty."

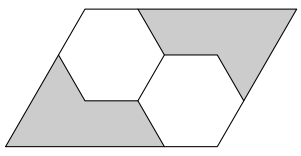
Manon: "At least two of us are innocent."

Mustafa: "At least two of us are guilty."

It turns out that those who are guilty lied, while those who are innocent told the truth.
How many of the five suspects are guilty?

A) 1 B) 2 C) 3 D) 4 E) 5

4. Two regular hexagons share a side and are situated inside a parallelogram as indicated in the figure. The area of the parallelogram equals 1.
What is the area of the two grey areas combined?



A) $\frac{1}{3}$ B) $\frac{2}{5}$ C) $\frac{5}{12}$ D) $\frac{3}{7}$ E) $\frac{1}{2}$

5. In the expression below, the ten dots are replaced by ten distinct digits (0 to 9) in such a way that none of the resulting two-digit numbers starts with 0:

$$\frac{\cdot\cdot+\cdot\cdot+\cdot\cdot}{\cdot\cdot-\cdot\cdot}$$

What is the largest possible outcome we can obtain?

A) $\frac{255}{7}$ B) $\frac{219}{2}$ C) 116 D) 222 E) 255

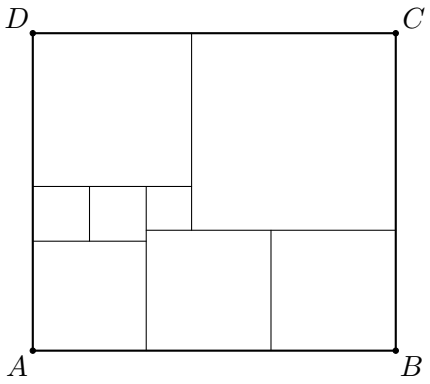
6. A 100×100 table is filled with numbers. The bottom left cell contains the number 0. For every other cell V, we consider a route from the bottom left cell to V, where in each step we go one cell to the right or one cell up (not diagonally). If we take the number of steps and add the numbers from the cells along the route, we obtain the number in cell V. In the figure, you see a partially filled table. The number 15, for example, is obtained as $4+(0+1+3+7)=15$.

⋮	⋮	⋮	⋰
3	7	15	⋯
1	3	7	⋯
0	1	3	⋯

What is the last digit of the number in the upper right cell of the 100×100 table?

A) 1 B) 3 C) 5 D) 7 E) 9

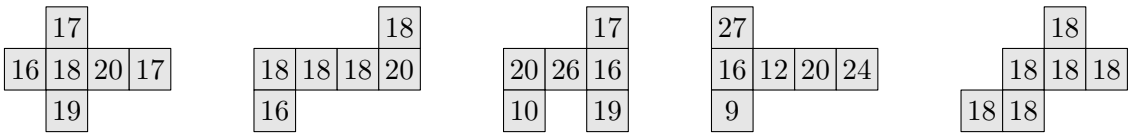
7. Rectangle $ABCD$ is divided into squares. The length of side AB is 16.



What is the length of side AD?

A) 13 B) $\frac{27}{2}$ C) 14 D) $\frac{29}{2}$ E) 15

8. Joep assigns the numbers 1 to 8 to the vertices of a cube (each vertex receiving a number different from the other vertices). For each face of the cube he adds the four numbers assigned to the vertices of that face and writes the resulting number on the face. Then, he cuts the cube open along some of the sides and flattens it out to obtain one of the five figures given below. Only one of these figures could represent Joep's cube.



Which figure could represent Joep's cube?

A) the first B) the second C) the third
D) the fourth E) the fifth

B-problems

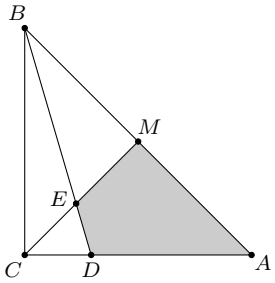
1. Isaac writes down a three digit number. None of its digits is a zero. Isaac gives his sheet with the number to Dilara, and below Isaac's number she writes down all three digit numbers that one can obtain by putting the digits of Isaac's number in a different order. Then she adds up all numbers on the sheet. The outcome is 1221.
What is the greatest number that Isaac could have written down?

2. There are two triples (a, b, c) of positive integers that satisfy the equations

$$\begin{aligned} ab + c &= 34, \\ a + bc &= 29. \end{aligned}$$

Which two triples are these?

3. Triangle ABC is an isosceles right angled triangle whose right angle is at C, with $|AC| = |BC| = 12$. Point M is the midpoint of side AB. A point D lies on side AC. Finally, point E is the intersection point of line segments CM and BD, see the figure.
If $|CD| = 3$, what is the area of quadrilateral $AMED$?



4. At a quiz you have to answer 10 questions. Each question is either difficult or easy. For a difficult question 5 points are being awarded for a correct answer and -1 point for an incorrect answer; for an easy question 3 points are being awarded for a correct answer and -1 point for an incorrect answer. Moreover, if you answer a question correctly, then the next question will be a difficult one; if you answer a question incorrectly, then the next question will be an easy one. You start with a difficult question.
How many distinct final scores are possible after 10 questions?