We eat problems for breakfast.

Preferably unsolved ones...

64th Dutch Mathematical Olympiad 2025

NEDERLANDSE WISKUNDE OLYMPIADE

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Introduction

The selection process for IMO 2026 in China started with a first round in January 2025, held locally at all participating schools. This first round consists of eight multiple choice questions and four open questions, to be solved within 2 hours. This year 6340 students from 321 secondary schools participated in the first round.

The 942 best students were invited to the second round, which was held at twelve universities throughout the country in March 2025. This round consists of five open questions, and two problems for which the students have to give extensive solutions and proofs. The contest lasts 2.5 hours.

The 124 best students were invited to the final round, together with some outstanding participants in the Kangaroo math contest or the Pythagoras Olympiad. In total, 153 students were invited. In the preceding months, we organize four training sessions at each of the universities to help them prepare for their participation in the final round.

The final, in September, contains five problems for which the students have to give extensive solutions and proofs. They are allowed 3 hours for this round. After the prizes had been awarded in the beginning of November, the Dutch Mathematical Olympiad concluded its 64th edition 2025.

The 33 most outstanding candidates of the Dutch Mathematical Olympiad were invited to an intensive seven-month training programme. The students met twice for a three-day training camp, three times for a single day, and finally for a six-day training camp in the beginning of June. They also worked on weekly problem sets to be sent in to a personal trainer by email.

In March a selection test of 3.5 hours was held to determine the four girls from the training group to represent the Netherlands in April at the EGMO in France, and to select the ten students from the training program which are sent to the Benelux Mathematical Olympiad (BxMO) held in April in the Netherlands. At the EGMO, the Dutch team won two silver and one bronze medals. For more information about the EGMO and the BxMO (including the 2026 paper), see www.egmo.org and www.bxmo.org.

In June the team for the International Mathematical Olympiad was selected by three team selection tests on three consecutive days, each lasting 4 hours. Five weeks later, the team had a training camp in Shanghai, China from July 5 to 13.

For younger students the Junior Mathematical Olympiad was held in June 2025 at the VU University Amsterdam. The students invited to participate in this event were the 100 best students of grade 1 and grade 2 of the popular

Kangaroo math contest. The competition consisted of two one-hour parts, one with eight multiple choice questions and one with eight open questions. The goal of this Junior Mathematical Olympiad is to scout talent and to stimulate them to participate in the first round of the Dutch Mathematical Olympiad.

We are grateful to Lance Bakker, Jinbi Jin and Raymond van Bommel for the composition of this booklet and the translation into English of most of the problems and the solutions.

Dutch delegation

The Dutch team for 67th edition of IMO 2026 in China consists of

- Roel Alkemade (18 years old)
  - honourable mention at BxMO 2025
  - bronze medal at IMO 2025
- Boaz Brouwer (15 years old)
  - bronze medal at BxMO 2026
- Josiah 't Hart (18 years old)
  - bronze medal at BxMO 2024, gold medal at BxMO 2025
  - bronze medal at IMO 2025
- Lisa Meij (18 years old)
  - bronze medal at EGMO 2025 and 2026
- Tobias Piet (17 years old)
  - participation at BxMO 2024, bronze medal at BxMO 2025 and 2026
  - observer C at IMO 2025
- Colin Smeets (16 years old)
  - participation at BxMO 2025

The team is coached by

- Quintijn Puite (team leader), Eindhoven University of Technology
- Johan Konter (deputy leader), Leiden University
- Kevin van Dijk (observer B), student Utrecht University
- Kees den Tex (observer B), student Leiden University

First Round, January 2025

Problems

A-problems

1. Five gnomes each have a red or blue hat on. The gnomes know that there are a total of three red hats and two blue hats. Gnome A looks at the hats of gnomes C and D and cannot deduce which colour hat he is wearing. Gnome B looks at the hats of gnomes C and E and also cannot deduce which colour hat he is wearing. Finally, gnome C looks at the hats of gnomes D and E and also cannot deduce which colour hat he is wearing.

Which of the following statements is definitely true?

A At least one of gnomes A and B is wearing a blue hat.
B At least one of gnomes A and B is wearing a red hat.
C Gnome C is wearing a red hat.
D At least one of gnomes D and E is wearing a blue hat.
E Gnomes D and E are both wearing a red hat.

2. The three sisters Alexa, Bente, and Charissa each have an age (in years) that consists of one digit, where Alexa is the youngest and Charissa the oldest. If you put Bente's age directly behind Alexa's, so that a two-digit number is formed, you get exactly the age of their father. Charissa's age behind Alexa's gives exactly the age of their mother. Charissa's age behind Bente's age gives their grandfather's age. Grandfather is 32 years older than father. If you add up the ages of father and mother, you get grandfather's age plus Bente's age.

How old is Alexa?

A 2 B 3 C 4 D 5 E 6

3. Laura is baking muffins and chocolate cake. In doing so, she uses exactly the same amount of sugar as flour. In the muffins, she puts twice as much flour as sugar. In the chocolate cake, she puts $\frac{5}{4}$ times as much sugar as flour.

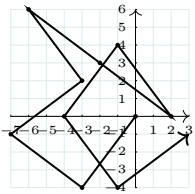
What proportion of flour does Laura use for the muffins?

A $\frac{1}{6}$ B $\frac{1}{3}$ C $\frac{1}{2}$ D $\frac{2}{3}$ E $\frac{5}{6}$

4. Of which of the following numbers is $2021 \cdot 2022 \cdot 2023 \cdot 2024 + 1$ the square?

A $2021 \cdot 2023 - 1$
B $2021 \cdot 2024 - 1$
C $2022 \cdot 2024 - 1$
D $2022 \cdot 2023 - 1$
E $2023 \cdot 2024 - 1$

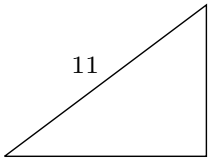
5. On a grid consisting of square boxes, a path from the origin is drawn by making one move at a time. A move consists of moving three squares horizontally while moving four vertically, or four horizontally while moving three vertically. The figure shows such a path, which consists of ten moves.



What is the minimum number of moves you need in order to reach the point $(1, 0)$ from the origin?

A 5 B 7 C 9 D 11 E 13

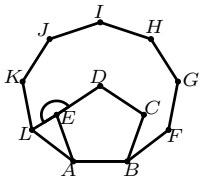
6. We consider a triangle with an angle of 90 degrees. The hypotenuse has length 11 and the perimeter of the triangle is 25.



What is the area of the triangle?

A 18 B $18\frac{1}{4}$ C $18\frac{1}{2}$ D $18\frac{3}{4}$ E 19

7. Given are a regular pentagon $ABCDE$ and a regular nonagon $ABFGHIJKL$ as in the figure. *Attention: the figure is not drawn to scale.*



What is the angle at E between the legs EL and ED?

A 176° B 177° C 178° D 179° E 180°

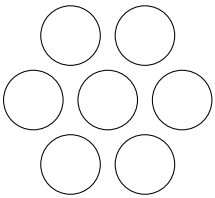
8. Babette puts a cross on the number line at exactly 1001 of the numbers 1 to 2025. Then for each pair of (not necessarily adjacent) crosses, she writes down the distance between them. Then she adds up all these distances.

In how many ways can she put her crosses to get the largest possible outcome?

A 1 B 2 C 1001 D 1025 E 2025

B-problems

1. Roel wants to surprise his sick friend Alan with a beautiful bouquet of three red and four white roses. He places a rose in the centre and the other six roses around it, as in the picture on the right. There are several ways to arrange these roses in a beautiful bouquet. We consider two ways the same if Roel can rotate the bouquet and get the other bouquet.

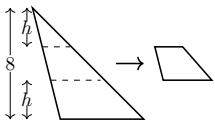


How many ways are there for Roel to arrange his bouquet?

2. On a digital clock, the possible times run from 00 : 00 : 00 to 23 : 59 : 59. It is possible to construct five times that together use each of the digits 0 to 9 exactly three times.

In doing so, what is the largest possible time difference between the earliest and latest time? *The time difference between, say,* 08 : 34 : 16 *and* 23 : 14 : 27 *is* 14 : 40 : 11.

3. From a triangle of height 8, parallel to the base, a piece of height h is cut away at both the top and bottom (see the illustration as an example). The area of the remaining piece has only one fifth of the area of the original triangle.



What is the height h of the pieces that are cut off? *Please note that the heights of the cut lines are not to scale.*

4. Lydia writes some one-digit numbers on the chalkboard and wants to add them all up. She does this as follows: first she adds the first two numbers, then she adds the result to the third number, and so on, until she has added all the numbers. After each addition, Lydia writes the (intermediate) answer on the board. For example, if Lydia starts with 3, 9, 5, then at the end the numbers 3, 9, 5, 12, 17 are written on the board (because $12 = 3 + 9$ and $17 = 12 + 5$): those are the digits 1, 2, 3, 5, 7, and 9. Lydia wants all digits 1 through 9 to appear on the chalkboard at least once when she is done. Moreover, she wants to start with as few numbers as possible.

Give an as short as possible sequence of numbers that Lydia could start with.

Solutions

A-problems

1. A At least one of gnomes A and B is wearing a blue hat.

2. B 3

3. B $\frac{1}{3}$

4. D $2022 \cdot 2023 - 1$

5. B 7

6. D $18\frac{3}{4}$

7. C 178°

8. D 1025

B-problems

1. 7

2. $23 : 57 : 33$

3. $3\frac{1}{5}$

4. 4, 5, 8, 6 or 5, 4, 8, 6 or 5, 8, 4, 9 or 6, 9, 8, 4 or 7, 8, 9, 6 or 8, 5, 4, 9 or 8, 7, 9, 6 or 9, 6, 8, 4

Second Round, March 2025

Problems

B-problems

1. Rowdy has a large rectangular sheet of paper whose sides have ratio $2025 : 1$. He cuts the paper into two equally sized rectangular pieces along the line connecting the midpoints of the long sides. Then he discards one of those two pieces, examines which sides of the remaining piece are now the long sides of the rectangle, and cuts that piece again into two equally sized rectangular pieces along the line connecting the midpoints of those long sides. He keeps repeating this process until he has cut a total of 100 times.

What is the ratio between the sides of the paper after 100 cuts?

2. A positive integer is called *rolling* if every two adjacent digits of this number differ by exactly 1. Examples of rolling numbers are 2101 and 9876.

How many 4-digit rolling numbers (not starting with a 0) are there?

3. Sam wants to colour each of the numbers 1 to 7 red or blue. For each number k from 1 to 3, if he colours k blue, then he must colour $2k$ and $2k + 1$ both red.

In how many ways can he colour the numbers 1 to 7 this way?

4. A rectangular jigsaw puzzle consists of corner pieces, edge pieces and centre pieces (all pieces not on a corner or at the edge). The pieces fit neatly into rows and columns. Both the number of pieces in the length and the number of pieces in the width do not exceed 50. The number of corner pieces plus the number of edge pieces is exactly 22% of the total number of pieces.

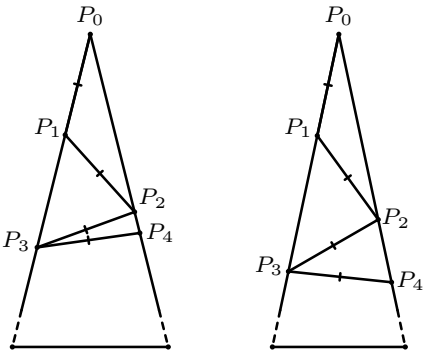
How many pieces does the whole puzzle consist of?

5. Triangle ABC is an isosceles right-angled triangle with a right angle at C. Point D is the midpoint of BC. The line through D perpendicular to AB intersects the extension of AC in E. The extension of AD intersects BE in G.

Determine the length $|AG|$ of the line segment AG if it is given that the area of triangle ABC equals 100.

C-problems

1. A monkey sits at the top P_0 of a very tall isosceles triangle with top angle α and a horizontal base. He always makes equally large jumps, landing alternately on the left and right sides of the triangle. He never jumps back to his previous position, but always to a position further from P_0 than the previous position on that side. If that is no longer possible, he stops jumping. The points at which the monkey successively lands are called $P_1, P_2, \ldots, P_n$. Below are two examples for $n = 4$.



(a) Suppose the monkey jumps four times ($n = 4$) and the last jump is horizontal, i.e. $|P_0P_3| = |P_0P_4|$. Calculate the top angle of the triangle.

(b) Suppose the top angle of the triangle is 6 degrees. What is the maximum number of times the monkey can jump? (The triangle is so high that he never reaches the ground.)

2. We consider the following equation in x and y:

$$(y + x^2 - 1)(y - x^2 + 1) = 4xy.$$

(a) Suppose $x = 10$. What are the possible values of y for which (x, y) is a solution of the equation?

(b) Suppose (x, y) is a solution of the equation where x is an integer. Prove that y or $-y$ is the square of an integer.

Solutions

B-problems

1. 2025 : 1024

2. 61

3. 41

4. 300

5. $6\sqrt{10}$

C-problems

1. (a) In triangle $P_0P_1P_2$, angle $\angle P_0P_2P_1$ is also equal to α and so angle $\angle P_0P_1P_2 = 180^\circ - 2\alpha$. Since angle $\angle P_0P_1P_3$ is straight, angle $\angle P_2P_1P_3$ equals 2α. We then find that $\angle P_1P_3P_2 = 2\alpha$ and $\angle P_1P_2P_3 = 180^\circ - 4\alpha$. Now we see that

$$\angle P_4P_2P_3 = 180^\circ - \angle P_0P_2P_1 - \angle P_1P_2P_3 = 180^\circ - \alpha - (180^\circ - 4\alpha) = 3\alpha.$$

Since triangle $P_2P_3P_4$ is isosceles, $\angle P_0P_4P_3 = 3\alpha$ and $\angle P_2P_3P_4 = 180^\circ - 6\alpha$. Finally, we now also see that

$$\angle P_0P_3P_4 = \angle P_0P_3P_2 + \angle P_2P_3P_4 = 2\alpha + (180^\circ - 6\alpha) = 180^\circ - 4\alpha.$$

So we see that triangle $P_0P_3P_4$ is exactly isosceles if $3\alpha = 180^\circ - 4\alpha$, or in other words if $\alpha = \frac{180}{7}^\circ$. □

(b) In part (a), we saw that after 4 jumps, the base angles of the last isosceles triangle are equal to 3α, so the apex angle of that last isosceles triangle is $180^\circ - 6\alpha$. So of the straight angle at P_3, exactly 6α remains. The previous isosceles triangle had base angles of 2α, so the next isosceles triangle (if it is still possible) has base angles of 4α. And this fifth step can be done as long as $4\alpha < 90^\circ$ because then we can form a new isosceles triangle with base angles 4α. If $4\alpha \geq 90^\circ$ this is not possible, because an isosceles triangle cannot have obtuse or right base angles.

By repeating this argument, we see that after n jumps, the base angles of the last isosceles triangle are equal to $(n-1)\cdot\alpha$ and another $(n+1)$-th jump can be made as long as $n\cdot\alpha < 90^\circ$. For $\alpha = 6^\circ$ we have $14\cdot 6^\circ = 84^\circ$ so there is still a 15th jump, but $15\cdot 6^\circ = 90^\circ$ so no more 16th jump can be taken. So the maximum number of jumps is 15. □

2. (a) For $x = 10$ the equation becomes

$$(y + 99)(y - 99) = 40y.$$

We can also write this as $y^2 - 40y - 99^2 = 0$. Since $99 = 9 \cdot 11$, we have $99^2 = 9^2 \cdot 11^2$. Because $9^2 = 81$ and $11^2 = 121$ have difference exactly 40, we see that the equation can be factored as follows: $(y+81)(y-121) = 0$. It follows that $y = 121$ or $y = -81$. □

(b) We are now going to solve the equation for a general integer x. This time we can rewrite the equation as

$$y^2 - (4x)y - (x^2 - 1)^2 = 0.$$

As in part (a), we note that $x^2 - 1 = (x - 1)(x + 1)$ hence

$$(x^2 - 1)^2 = (x - 1)^2(x + 1)^2 = (x^2 - 2x + 1)(x^2 + 2x + 1).$$

We see that the factors differ by $4x$. So the equation factors as

$$\left(y + (x - 1)^2\right)\left(y - (x + 1)^2\right) = 0.$$

This means that either $y = -(x-1)^2$ or $y = (x+1)^2$. In all cases either y or $-y$ is the square of an integer. □

Final Round, September 2025

Problems

1. Behind three of the 25 squares of a scratch card with 5×5 squares a prize is hidden. It is only known that those three squares are all next to each other and together form a 3×1-rectangle (horizontally or vertically). You pay for the scratch card an amount of euros that is equal to the number of squares you are allowed to scratch open (so for 6 euros you are allowed to scratch open 6 squares).

What is the least amount of euros you have to pay for the scratch card to make sure you win a prize?

2. Consider the equation

$$n! - (n-1)n = 4p^2$$

where n is a positive integer and p is a prime number. (Here $n!$ represents the product of the numbers 1 to n. For example, $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$.)

Find all solutions (n, p).

3. Amber has a square consisting of 10×10 square boxes. One at a time, she randomly chooses one of the hundred squares and fills it with a number. The number she puts in a square is always equal to the total number of squares already filled in that square's row and column. So in the first square she chooses she writes a 0, in the second square a 0 or a 1, depending on whether it is in a different row and a different column or not, et cetera. So in each square there will be a number from 0 to 18 inclusive.

After Amber has written a number in each square, she adds up all the numbers. What is/are the possible result(s) of this addition?

4. Within a triangle ABC lies a point P with the property that $\angle ACP = \angle BAP$ and $\angle BCP = \angle ABP$.

Prove that P lies on the median of triangle ABC from vertex C. (The median is the line passing through vertex C and the midpoint of side AB.)

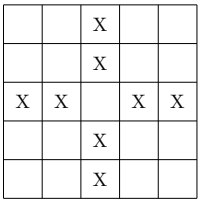
5. The vertices of a regular n-gon are numbered clockwise from 0 to $n-1$. We have n pawns, also numbered from 0 to $n-1$, and we put one pawn on each vertex. For each pawn, we consider how many vertices it has to move clockwise to get to the vertex with its own number. We want to place the pawns such that those n numbers are different.

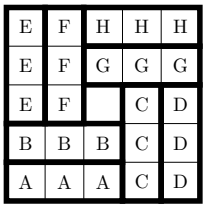
Determine all n for which this is possible.

Solutions

1. You can always find a prize by scratching open the eight boxes where there is a cross in the figure at the bottom left. Indeed, it it is easy to verify that each of the 30 possible 3×1-rectangles overlaps with at least one of the boxes marked with a cross.

On the other hand, it is really necessary to scratch open eight boxes. In the figure below right, you can see eight possibilities for the three prize boxes: A to H. If seven or fewer boxes were scratched open, then in at least one of these eight possible situations you would not have found a prize.





□

2. If we substitute $n = 1, \ldots, 5$, we see that on the left side we get the numbers $1, 0, 0, 12$ and 100 respectively. The number on the right-hand side is a positive multiple of 4, so 1 and 0 are not possible. Furthermore, we see that $12 = 4 \cdot 3$ and 3 is not a square. Finally, we see that $100 = 4 \cdot 25$ and 25 is the square of the prime number $p = 5$. So the only solution with $1 \leq n \leq 5$ is $(n, p) = (5, 5)$.

Suppose (n, p) is a solution with $n \geq 6$. First, we note that

$$n! - (n-1)n = ((n-2)! - 1)(n-1)n$$

is divisible by $n - 1$ and n. Only one of those two numbers can be divisible by p, because their difference, which is 1, is not divisible by p. The divisors of $4p^2$ are $1, 2, 4, p, 2p, 4p, p^2, 2p^2$ and $4p^2$ and only the divisors 1, 2 and 4 are not divisible by p (except if $p = 2$, then only the divisor 1 remains). So that means $n - 1$ or n must equal 1, 2 or 4. In particular, it follows that n is at most 5, which is a contradiction. □

3. We will prove that the result of this addition is always 900.

In each square, write two more numbers: in yellow the total number of squares already filled in the row of that square, and in red the total number of squares already filled in the column of that square. The number Amber writes down is then the yellow number plus the red number. If you look at the yellow numbers per row, you will see that you get the numbers 0 up to 9: the first square chosen gets a 0, the next one a 1, and so on. Similarly, you

get the red numbers 0 to 9 per column. So the sum of the yellow numbers in a row (and the red numbers in a column) is

$$0 + 1 + 2 + \ldots + 9 = 45.$$

In total, there are 10 rows and 10 columns. So the total sum is always $20 \cdot 45 = 900$. □

4. Suppose D is the intersection of CP and AB. Then, in triangles ADP and CDA, two angles are equal and so the triangles are similar. It follows that $\frac{|AD|}{|DP|} = \frac{|CD|}{|AD|}$. Similarly, triangles BDP and CDB are similar and we find that $\frac{|BD|}{|DP|} = \frac{|CD|}{|BD|}$. Combining all this, we find that

$$|AD|^2 = |CD| \cdot |DP| = |BD|^2.$$

So we find that $|AD| = |BD|$ and thus D is the midpoint of the line segment AB. Therefore CP is the median of triangle ABC through C. □

5. We show that this is possible if and only if n is odd.

First suppose that n is odd. A possible solution is then given as follows. Put pawn 0 at vertex 0 and for every $i = 1, \ldots, n-1$, put pawn i at vertex $n - i$.

If $0 < i \leq \frac{n-1}{2}$, then i is smaller than $n - i$ and pawn i, which starts at vertex $n - i$, must therefore first move i places to get to vertex 0 and then another i to end at the correct vertex. So the number of spots these pawns must move up is $2, 4$, and so on, up to $2 \cdot \frac{n-1}{2} = n - 1$. Pawn 0 must move up 0 spots. Altogether, those are exactly all possible even numbers.

If $\frac{n+1}{2} \leq i < n$, then $n - i$ is smaller than i and pawn i, starting at vertex $n - i$, must therefore move up $i - (n - i) = 2i - n$ steps clockwise to arrive at the correct vertex. So these pawns must move 1, 3, 5, and so on, up to $n - 2$, spots to arrive at the correct vertex. Those are exactly the odd numbers.

Now suppose that we have a solution. We are going to show that n must be odd. Suppose pawn i starts at vertex a_i. The distance to the correct vertex, vertex i, is then $\Delta_i = i - a_i$ if $i \geq a_i$, and $\Delta_i = (n + i) - a_i$ if $i < a_i$. On the one hand, the pawns all start at different vertices and so we have that

$$\sum_{i=0}^{n-1} a_i = \sum_{j=0}^{n-1} j.$$

On the other hand, the pawns also all end at different vertices:

$$\sum_{i=0}^{n-1} i = \left(\sum_{i=0}^{n-1} (a_i + \Delta_i) \right) - cn,$$

where c is the number of i for which $i < a_i$. In particular, it therefore follows that

$$\sum_{i=0}^{n-1} \Delta_i = cn.$$

If the Δ_i are all distinct, then they are exactly the numbers 0 up to $n-1$. So we then find that $0 + 1 + \ldots + (n-1) = cn$, or equivalently $\frac{1}{2}n(n-1) = cn$. This implies that $\frac{1}{2}(n-1) = c$, so $n = 2c+1$. Hence, n is odd, as desired. $\square$

BxMO/EGMO Team Selection Test, March 2026

Problems

1. Let $ABCD$ be a rectangle such that $|AB| < |BC|$, and let Γ be its circumcircle. Let P be a point on the arc BC of Γ not containing A and let Q be a point on the arc CD of Γ not containing A, such that $|BP| = |CQ|$. Let X be the intersection of AP with the line through B perpendicular to AQ.

Prove that $|PQ| = |XQ|$.

2. Determine all integers z that are of the form $\frac{a^2-b^2}{b}$ with a and b are positive integers.

3. Kees is studying the weather. Every day, he checks whether it rains or not. On day 1 it is certain that it does rain, on day 2 it is certain it doesn't rain. From day 3 onwards, at the end of every day, Kees writes down what fraction of the days it has rained. For example, if there was rain on day 3, but not on day 4 and 5, the first three numbers that Kees writes down are $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{2}{5}$.

So days 1 and 2 are fixed, but depending on what happens on the days after that, different sequences may be obtained in this way. We say that a number $c \in (0, 1)$ is *indispensible* if the following holds: every sequence that is obtained in this way, that has a term smaller than c and has a term larger than c, must also contain the number c itself as a term.

Determine all indispensible numbers $c \in (0, 1)$.

4. Let $\triangle ABC$ be an acute triangle with $|BC| > |AC|$ and centroid Z. Let M be the midpoint of AB and let F be the foot of the altitude from C. Let P be the second intersection of the median CM with the circumcircle Γ of $\triangle ABC$. Let Q be the intersection of ZF and Γ such that F lies between Z and Q. Show that $FMPQ$ is a cyclic quadrilateral.

5. Consider the sequence $a_0, a_1, a_2, \ldots$ defined by $a_0 = a_1 = 1$ and

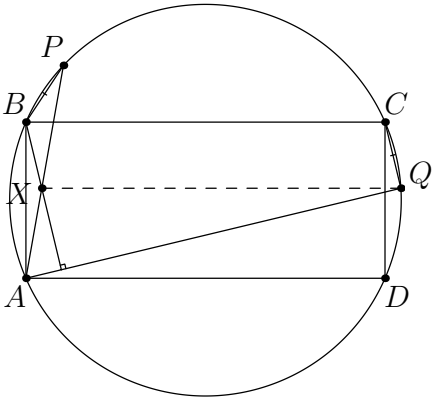
$$a_{n-1}^2 = a_{n-2}a_n - 2$$

for all integers $n \geq 2$.

Show that every term in this sequence is an integer.

Solutions

1. As $|BP| = |CQ|$, both chords subtend the same angles, so in particular $\angle BAP = \angle CDQ$. As AB and CD are parallel, it follows that $AP \parallel DQ$. By Thales's Theorem (as BD is a diameter of Γ), we have $BQ \perp DQ$ and therefore $AP \perp BQ$.



As X lies on AP, we have $AX \perp BQ$ and $BX \perp AQ$. It follows that X is the orthocentre of triangle $\triangle ABQ$. Therefore we have $AB \perp QX$ as well.

As $ABCD$ is a rectangle and $AB \perp QX$, it follows that $QX \parallel AD$. Since $AP \parallel DQ$, it also follows that $ADQX$ is a parallellogram and that $ADQP$ is an isosceles trapezoid. Hence $|PQ| = |AD| = |XQ|$. □

2. We show that every integer z except for $z = \pm 1, \pm 2, \pm 4$ can be written in the given form.

We start by showing that every z expect for possibly $z = \pm 1, \pm 2, \pm 4$ can be written in the given form. First note that $0 = \frac{1^2-1^2}{1}$, $8 = \frac{3^2-1^2}{1}$, $-8 = \frac{3^2-9^2}{9}$, so 0, 8, and -8 can all be written in the given form.

Note that every odd integer $z \geq 3$ can be written in this form: write $z = 2n+1$ with $n \geq 1$ and let $a = n(n+1)$ and $b = n^2$. As $n \geq 1$, we have $a, b \geq 1$ as well, so

$$\frac{n^2(n+1)^2 - n^4}{n^2} = (n+1)^2 - n^2 = 2n + 1 = z.$$

Also note that every odd integer $z \leq -3$ also can be written in this form: write $z = -2n - 1$ with $n \geq 1$ and let $a = n(n+1)$ and $b = (n+1)^2$. As $n \geq 1$, we have $a, b \geq 1$ as well, so

$$\frac{n^2(n+1)^2 - (n+1)^4}{(n+1)^2} = n^2 - (n+1)^2 = -2n - 1 = z.$$

Observe that if z can be written as $z = \frac{a^2-b^2}{b}$ with a and b are positive integers, then so can $2z = \frac{(2a)^2-(2b)^2}{2b}$. It follows that every integer except for possibly $z = \pm 1, \pm 2, \pm 4$ are of the given form.

Finally, we show that $z = \pm 1, \pm 2, \pm 4$ cannot be written in the given form. First suppose that $z = \pm 1$. If $z = \frac{a^2-b^2}{b}$, then $b(b \pm 1) = a^2$. Since $\gcd(b, b \pm 1) = 1$, it follows that both b and $b \pm 1$ are squares. As the only consecutive squares are 0 and 1, it follows that a must be zero. Hence in this case z is not of the given form.

Next, suppose that $z = \pm 2$. If $z = \frac{a^2-b^2}{b}$, then $b(b \pm 2) = a^2$. If b is even, then so is a, and dividing each by 2 shows that $\frac{1}{2}z = \pm 1$ is of the given form, contradicting the previous case. If b is odd, then $\gcd(b, b \pm 2) = 1$, so b and $b \pm 2$ would have to be squares with difference 2. However, no such pair of squares exist, so z cannot be of the given form.

Finally, suppose that $z = \pm 4$. If $z = \frac{a^2-b^2}{b}$, then $b(b \pm 4) = a^2$. If b is even, then so is a, and dividing each by 2 shows that $\frac{1}{2}z = \pm 2$ is of the given form, contradicting the previous case. If b is odd, then $\gcd(b, b \pm 4) = 1$, so b and $b \pm 4$ would have to be squares with difference 4. As the only consecutive squares are 0 and 4, it follows that a must be zero. Hence in this case z is not of the given form either. □

3. We show that the indispensible numbers are $\frac{1}{2}$, $\frac{1}{n}$ with $n \geq 3$, and $1 - \frac{1}{n}$ with $n \geq 3$. We first show that all other numbers are not indispensible.

If it stops raining starting from day 3, then the sequence obtained is $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$. Every number smaller than $\frac{1}{3}$ and not of the form $\frac{1}{n}$ is not a term of this sequence and therefore are not indispensible. Repeating the argument for the situation in which it rains on every day from day 3 onwards, we see that every number greater than $\frac{2}{3}$ not of the form $1 - \frac{1}{n}$ is not indispensible.

If it rains on day 3 and it doesn't rain on all later days, we obtain the sequence $\frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \ldots$. None of the numbers between $\frac{1}{2}$ and $\frac{2}{3}$ are terms of this sequence, so none of those are indispensible. In the same way, if it doesn't rain on day 3 and it does rain on all later days, we obtain the sequence $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \ldots$. Then none of the numbers between $\frac{1}{3}$ and $\frac{1}{2}$ are terms of this sequence, so none of those are indispensible either.

This shows that the only possibly indispensible numbers are those of the form $\frac{1}{n}$ or of the form $1 - \frac{1}{n}$. We now show that each of these are indeed indispensible.

First consider the case $c = \frac{1}{n}$ with $n \geq 3$. Suppose we obtain a sequence containing a term greater than c and one less than c. The first term of the sequence is either $\frac{1}{3}$ or $\frac{2}{3}$, both of which are at least $\frac{1}{n}$. So there must be a term $\frac{a}{b}$ of the sequence that is at least $\frac{1}{n}$, such that the following term is less than $\frac{1}{n}$.

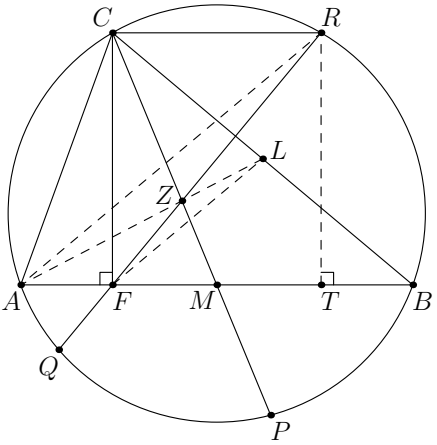
Suppose for a contradiction that $\frac{a}{b} > \frac{1}{n}$. Then $an > b$, or equivalently $an \geq b+1$. Note that the following term in the sequence must be either $\frac{a+1}{b+1}$ or $\frac{a}{b+1}$. As it is also smaller than $\frac{a}{b}$, it must therefore be $\frac{a}{b+1}$. Hence $\frac{a}{b+1} < \frac{1}{n}$, so $b+1 > an$ and we therefore find that $an \geq b+1 > an$, which is a contradiction. Therefore $c = \frac{a}{b}$ is itself a term of the sequence.

Now consider the case $c = 1 - \frac{1}{n}$ with $n \geq 3$ and suppose we obtain a sequence containing both a term greater than c and one less than c. In the same way as in the previous case, we see that there is a term $\frac{a}{b} \leq 1 - \frac{1}{n}$ for which the following term is greater than $1 - \frac{1}{n}$ – and that this following term must be $\frac{a+1}{b+1}$. Suppose again for a contradiction that $\frac{a}{b} < 1 - \frac{1}{n}$. Then $\frac{1}{n} < \frac{b-a}{b}$, from which follows that $n(b-a) > b$ and therefore that $n(b-a) \geq b+1$. But on the other hand, from $\frac{a+1}{b+1} > 1 - \frac{1}{n}$ follows that $\frac{1}{n} > \frac{b-a}{b+1}$ and therefore that $b+1 > n(b-a)$. So $b+1 > n(b-a) \geq b+1$, which is a contradiction. Therefore $c = \frac{a}{b}$ is itself a term of the sequence.

Finally consider the remaining case $c = \frac{1}{2}$. Assuming without loss of generality that it rains on day 3, we can repeat the argument of the $c = \frac{1}{n}$ with $n \geq 3$ case to show that $\frac{1}{2}$ has to be a term of any obtained sequence which also contains a term that is less than $\frac{1}{2}$.

Therefore the indispensible numbers are $\frac{1}{2}$, $\frac{1}{n}$ with $n \geq 3$, and $1 - \frac{1}{n}$ with $n \geq 3$. □

4. Let L be the midpoint of BC. Let R be the point such that $RC \parallel AB$ and $RA \parallel LF$.



We first show that R lies on Γ. By angle chasing, $\angle ARC = \angle FRC - \angle FRA = \angle RFM - \angle RFL = \angle LFM$. Moreover, $\angle LFM = \angle ABC$ as by Thales's Theorem on $\triangle CFB$, the triangle $\triangle LFB$ is isosceles. Therefore R lies on Γ.

As ML is a midsegment of $\triangle ABC$, we have $\angle RCA = 180° - \angle CAB = 180° - \angle LMB = \angle FML$. As also $\angle ARC = \angle LFM$, it follows that $\triangle RCA \sim$

$\triangle FML$. As Z is the centroid of $\triangle ABC$, we have $|CZ| : |ZM| = |AZ| : |ZL| = 2 : 1$.

Consider the homothety with centre Z and scale $-\frac{1}{2}$. This maps A to L and C to M and therefore must map R to F (as the similarity $\triangle RCA \sim \triangle FML$ preserves orientation). Hence we see that R lies on ZF and that the ratio $|RZ| : |ZF|$ equals $2 : 1$.

Finally, we apply the power theorem. Using the ratio's $|CZ| : |ZM| = |RZ| : |ZF| = 2 : 1$ it follows that, as directed lengths:

$$ZM \cdot ZP = -\tfrac{1}{2}ZC \cdot ZP = -\tfrac{1}{2}ZR \cdot ZQ = ZF \cdot ZQ.$$

By the power theorem, it follows that $FMPQ$ is a cyclic quadrilateral. $\square$

5. First note that the sequence is increasing and therefore positive, starting from the term for $n = 2$. Indeed, from $a_{n-1} \geq a_{n-2} > 0$ it follows by induction that $a_n = \frac{a_{n-1}^2+2}{a_{n-2}} > \frac{a_{n-1}^2}{a_{n-2}} \geq a_{n-1}$.

Adding the recursion relation $a_{n-1}^2 = a_{n-2}a_n - 2$ to that of the following recursion step $a_{n-1}a_{n+1} - 2 = a_n^2$, we obtain

$$a_{n-1}^2 + a_{n-1}a_{n+1} = a_{n-2}a_n + a_n^2.$$

As $a_{n-1}a_n$ is positive, we may divide both sides of the equality by this number:

$$\frac{a_{n-1} + a_{n+1}}{a_n} = \frac{a_{n-2} + a_n}{a_{n-1}}.$$

Writing c_n for $\frac{a_{n-1}+a_{n+1}}{a_n}$, we obtain $c_n = c_{n-1}$. Therefore the sequence $(c_n)_{n\geq 1}$ is constant. Computing $c_1 = \frac{a_0+a_2}{a_1} = \frac{1+3}{1} = 4$, we see that $\frac{a_{n-1}+a_{n+1}}{a_n} = c_n = 4$ for all $n \geq 1$, and therefore that

$$a_{n+1} = 4a_n - a_{n-1}.$$

As a_0 and a_1 are integers, it follows by induction that all a_n are integers. $\square$

IMO Team Selection Test 1, June 2026

Problems

1. Let H be the orthocentre of an acute triangle $\triangle ABC$ and let D be the foot of the altitude from A. The circle with centre A and radius $|AD|$ is denoted by Ω. We draw the tangents from B and C to Ω distinct from BC and define P as their intersection. Prove that the radius of the incircle of $\triangle BCP$ is equal to $|HD|$.

2. Let d be a positive integer. Define the sequence $a_1, a_2, a_3, \ldots$ by $a_1 = 1$ and

$$a_{n+1} = n\lfloor \tfrac{a_n}{n} \rfloor + d \quad \text{for all } n \geq 1.$$

Prove that there exists a positive integer M such that $a_M, a_{M+1}, a_{M+2}, \ldots$ is an arithmetic progression.

3. Let $n \geq 3$ be a given integer and let $S \geq 1$ a multiple of $10^n - 3$. Suppose that the decimal representation of S consists of m digits with $n < m < 2n-1$. Prove that the decimal representation of S contains at least 3 distinct digits.

For example, $10^4 - 3 = 9997$ *only consists of 2 distinct digits.*

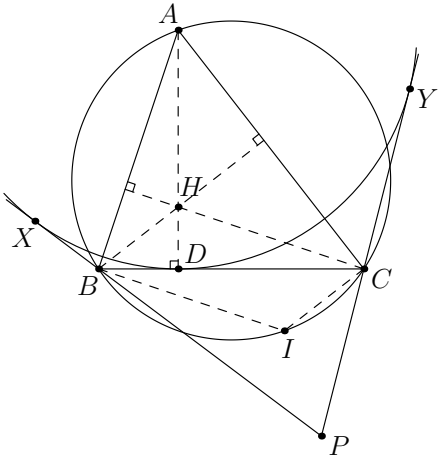
4. Kees and Lars play the following game where they colour the points of the interval $[0, 1]$ white or black. Each turn consists of two phases: first, one player chooses a number $\ell \in [0, 1]$, then the other chooses an interval $[a, b]$ of length $b - a = \ell$ in $[0, 1]$ and paints each point of it in the other colour than its current colour. In the next turn, the roles are reversed.

Initially, the entire interval $[0, 1]$ is white. Kees starts by choosing a number ℓ. After 2026 turns, the total length W of white intervals is calculated. Kees wins if $W \leq \frac{1}{2}$ and Lars wins if $W > \frac{1}{2}$.

Which player has a winning strategy?

Solutions

1. Let I be the incentre of $\triangle BCP$. Note that $BC \perp DA$, so BC is tangent to Ω. Since PB and PC are also tangent to Ω, this is the excircle of $\triangle BCP$ opposite to P. It follows that BA is the external bisector of $\angle PBC$. Since BI is the internal bisector of that angle, we find that $BI \perp AB$.



Similarly, $CI \perp AC$, so by Thales's Theorem I is the antipode of A in the circumcircle of $\triangle ABC$. This antipode is alternatively characterized as the reflection of H in the midpoint of BC. It now immediately follows that the distance from I to BC is equal to the distance from H to BC, which is what we wanted to prove. $\square$

2. We first prove by induction on n that $a_n \leq dn$. For $n = 1$ we indeed find that $a_1 = 1 \leq d$. Now suppose that $a_n \leq dn$. Then by the recurrence relation we have $a_{n+1} = n\lfloor \frac{a_n}{n} \rfloor + d \leq n\lfloor \frac{dn}{n} \rfloor + d = nd + d = d(n+1)$.

Now we prove that $b_n := \lfloor \frac{a_n}{n} \rfloor$ is eventually constant. For this we first note that we have just proved that $b_n = \lfloor \frac{a_n}{n} \rfloor \leq \lfloor \frac{dn}{n} \rfloor = d$. However, from the recurrence relation we also find that

$$\frac{a_{n+1}}{n+1} = \frac{nb_n + d}{n+1} \geq \frac{nb_n + b_n}{n+1} = \frac{(n+1)b_n}{n+1} = b_n.$$

Because the right side is an integer, it follows that $b_{n+1} = \lfloor \frac{a_{n+1}}{n+1} \rfloor \geq b_n$. Now we have a non-decreasing sequence of integers that is bounded from above (by d). This implies that b_n is eventually constant, say for all $n \geq M$ we have $b_{n-1} = c$. We conclude that we can write the sequence for $n \geq M$ as the arithmetic progression $a_n = (n-1)b_{n-1} + d = (n-1)c + d = cn + (d-c)$. $\square$

3. Write $S = \overline{a_m \dots a_2 a_1}$ in the decimal system. Let $d = 10^n - 3$ and k be such that $S = kd$. Then we have

$$3 \cdot \overline{a_m \dots a_{n+1}} + \overline{a_n \dots a_2 a_1} \equiv 10^n \cdot \overline{a_m \dots a_{n+1}} + \overline{a_n \dots a_2 a_1} = S \equiv 0 \pmod{d}.$$

Because the left hand side is positive, it must be greater than or equal to d. From this it follows that

$$\overline{a_n \dots a_2 a_1} \geq d - 3 \cdot \overline{a_m \dots a_{n+1}} \geq d - 3 \cdot (10^{n-2} - 1) > 9 \cdot 10^{n-1}.$$

This means that $a_n = 9$.

On the other hand, we have $S = kd = k \cdot 10^n - 3k = (k-1)10^n + (10^n - 3k)$. So $\overline{a_m \dots a_{n+1}} = k - 1$ and $\overline{a_n \dots a_2 a_1} = 10^n - 3k$. This implies that $a_{n+1} \equiv k - 1 \pmod{10}$ and $a_1 \equiv -3k \pmod{10}$. Because $k - 1 \equiv -3k$ has no solution modulo 10 (otherwise it would also have a solution modulo 2), we conclude that $a_{n+1} \neq a_1$.

Now suppose for a contradiction that S only consists of two distinct digits. Then we must have either $a_{n+1} = 9$ or $a_1 = 9$. In the first case we find that $k \equiv a_{n+1} + 1 \equiv 0 \pmod{10}$, so the other digit is $a_1 \equiv -3 \cdot 0 = 0 \pmod{10}$. In the second case we find $k \equiv 9/(-3) = -3 \equiv 7 \pmod{10}$, so the other digit is $a_{n+1} \equiv k - 1 \equiv 7 - 1 = 6 \pmod{10}$.

In both cases it follows from the sum of the digits that S is divisible by 3, and similarly that $k - 1 = \overline{a_m \dots a_{n+1}}$ is divisible by 3. But $d = 10^n - 3 \equiv 1 \pmod{3}$ and $k \equiv 1 \pmod{3}$. Thus $S = kd$ cannot be divisible by 3. We conclude that S has at least three distinct digits. □

4. Lars has a winning strategy by ensuring that after every even turn at least one of the two ends of the interval $[0, 1]$ is white. We define W_n as the total length of white intervals after the n-th turn. In particular, we are given that $W_0 = 1$. We now prove the following claim which ensures that Lars wins.

Claim. *Lars can guarantee that after every even turn $W_{2k} > \frac{1}{2}$ and that for a certain $\delta > 0$ at least one of the open intervals $(0, \delta)$ or $(1 - \delta, 1)$ is completely white.*

We prove this claim by induction. As a base case we have "after the zeroth turn", i.e. at the start of the game. Then the entire interval $[0, 1]$ is white, which is definitely more than half of the interval. Now suppose that after the $2k$-th turn we have the situation described in the claim.

If Kees chooses $\ell = 0$ or $\ell = 1$ in the $(2k + 1)$-st turn, then Lars can choose the same ℓ in the $(2k+2)$-nd turn. This causes $[0, 1]$ to have the same colours after these two turns, with the exception of at most two points in the case that $\ell = 0$.

Suppose Kees instead chooses $0 < \ell < 1$ in the $(2k+1)$-st turn. As there is at most one black end, Lars can place the segment in such a way that for a certain $\delta > 0$, the intervals $(0, \delta)$ and $(1-\delta, 1)$ at both ends are white after this turn. In case one of the ends is black, Lars can place the segment over it. In the other case, i.e. if both ends are white, then Lars can place the segment in the middle of $[0, 1]$.

If now $W_{2k+1} > \frac{1}{2}$, then Lars subsequently chooses $\ell = 0$ in the $(2k+2)$-nd turn, and we get that $W_{2k+2} = W_{2k+1} > \frac{1}{2}$. Now suppose that $W_{2k+1} \leq \frac{1}{2}$. We already had that $(0, \delta)$ and $(1-\delta, 1)$ are the white ends of the interval $[0, 1]$ that Lars has guaranteed after the $(2k+1)$-st turn. Then for the $(2k+2)$-nd turn Lars chooses an ℓ such that $1 - \delta < \ell < 1$, for example $\ell = 1 - \delta/2$. Kees cannot place this interval such that both ends become black, since $\ell < 1$. And wherever Kees places the interval, its complement will be completely white (with the possible exception of the endpoints 0 and 1), with a total length of $1 - \ell$. This means that the interval covers all black intervals, which together have a total length of $1 - W_{2k+1} \geq \frac{1}{2}$. After the $(2k+2)$-nd turn, therefore, we have $W_{2k+2} = (1-\ell) + (1 - W_{2k+1}) > \frac{1}{2}$ with at least one of the ends being white. □

IMO Team Selection Test 2, June 2026

Problems

1. Five real numbers are placed in a circle. Each number is 1 less than the product of its two neighbours. The sum of the five numbers is 0. What are the possible values of the product of the five numbers?

You may assume that at least one such circle of numbers exists.

2. Let n be a positive integer. We call a set A of positive integers *fivish* if the following properties hold:

- all numbers in A consist of n digits in their decimal representation,
- the numbers in A contain no zeros and no fives in their decimal representation,
- for all pairs $a, b \in A$ with $b > a$, the difference $b - a$ always contains a five in its decimal representation.

What is the maximum number of elements a fivish set can contain (in terms of n)?

3. Let n be a positive integer. On an $n \times n$ board, n stones are placed such that each stone occupies one square and each square contains at most one stone. For which n is it possible that for all $1 \leq i, j \leq n$ the number of stones in the i-th row is divisible by i and the number of stones in the j-th column is divisible by j?

4. Let $\triangle ABC$ be an acute triangle and D a point on segment BC. Consider points E and F on line AD such that $EB \perp AB$ and $FC \perp AC$, and consider points G and H on line BC such that EG is parallel to AC and such that FH is parallel to AB. The circumcircle of $\triangle BGE$ intersects line AB for the second time in M, and the circumcircle of $\triangle CHF$ intersects AC for the second time in N. Prove that MG, NH and AD are concurrent.

Solutions

1. We name the numbers in (cyclic) order a_0, a_1, a_2, a_3, a_4 so that the given conditions translate to $a_{n-1}a_{n+1} = a_n + 1$ for all n (with indices read modulo 5) and $a_0 + a_1 + a_2 + a_3 + a_4 = 0$. The requested product is then equal to $a_0a_1a_2a_3a_4$.

We rewrite this product in a clever way:

$$\begin{aligned} a_0a_1a_2a_3a_4 &= (a_0a_2)(a_1a_3)a_4 = (a_1+1)(a_2+1)a_4 \\ &= (a_1+1)(a_2a_4 + a_4) = (a_1+1)((a_3+1)+a_4) \\ &= (a_1a_3 + a_1 + a_1a_4) + (a_3 + 1 + a_4) \\ &= (a_2+1) + a_1 + (a_0+1) + a_3 + 1 + a_4 \\ &= (a_0 + a_1 + a_2 + a_3 + a_4) + 3 = 0 + 3 \\ &= 3. \end{aligned}$$

Therefore the requested product equals 3. □

2. The maximum number of elements a fivish set can contain is 2^n.

We first show that this number can be attained. Choose for A_n the set of numbers consisting of n digits, each of which are either 1 or 6. There are 2^n such numbers and they contain no zeros and fives. If we now take two distinct numbers in A_n, we consider the first position k from the right where they differ. We can write the numbers as $10^k \cdot x + 6 \cdot 10^{k-1} + c$ and $10^k \cdot y + 1 \cdot 10^{k-1} + c$. The difference is

$$10^k \cdot (x-y) + 5 \cdot 10^{k-1} \qquad \text{or} \qquad 10^k \cdot (y-x-1) + 5 \cdot 10^{k-1}.$$

This ends with a five followed by k zeros. The set A_n is therefore fivish.

Now we will show that no fivish set contains more than 2^n elements. For this we call the digits 1, 2, 3 and 4 *low* and the digits 6, 7, 8, 9 *high*. Suppose we have a fivish set. We can assign to each element of this set a high-low sequence (for example, 1793 becomes LHHL). We claim that the difference between two numbers with the same high-low sequence never contains the digit five. From this claim it will follow that the set contains at most one element for each high-low sequence of length n. As there are 2^n possible high-low sequences, the set contains at most 2^n elements.

Now we prove the claim. We take two numbers with the same high-low sequence, and calculate the difference using the "normal" method: from right to left, occasionally borrowing a 1 from the next position. At each position we calculate the difference between two digits in the same category (low or high), and then possibly subtract 1 that we borrowed for the previous position. The possible outcomes are thus $(1-4)-1 = (6-9)-1 = -4$ up to $(4-1) = (9-6) = 3$, which corresponds to the digits 6, 7, 8, 9, 0, 1, 2, 3. Therefore the digit 5 cannot occur in the outcome, which proves the claim. □

3. All positive integers n except 2, 5 and 9 are possible.

Suppose $n = 2$. If row 2 (resp. column 2) contains stones, then by counting per column (resp. row) there are at least $1 + 2 = 3$ stones. Otherwise, row 2 and column 2 are both empty and there is at most 1 stone.

Suppose $n = 5$. If any row (resp. column) with number at least 3 contains stones, then by counting per column (resp. row) there are at least $1+2+3 = 6$ stones. Otherwise, all rows and columns with numbers at least 3 are empty and you have at most 4 stones.

Suppose $n = 9$. If any row (resp. column) with number at least 4 contains stones, then by counting per column (resp. row) there are at least $1+2+3+4 = 10$ stones. Otherwise, all rows and columns with numbers at least 4 are empty and you have at most 9 stones. In that case you must completely fill the 3×3-subboard with stones. But then there are 3 stones in row 2, while 3 is not divisible by 2.

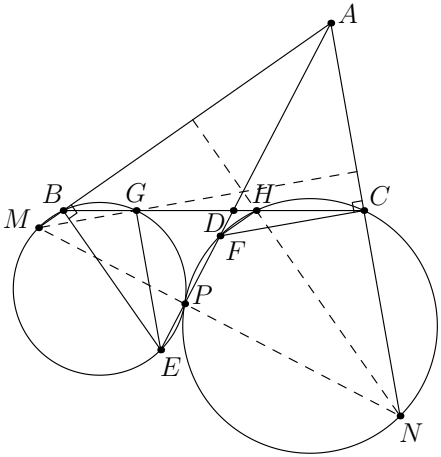
Now we will prove that all other n are possible. First note that the triangular numbers are easy to construct. Namely, suppose that $n = \frac{k(k+1)}{2}$ for some k, then for $1 \leq i, j \leq k$ you can place a stone on square (i, j) whenever $i + j \geq k + 1$.

Now suppose that n is not a triangular number. Let $\frac{k(k+1)}{2}$ be the largest triangular number smaller than n. If we define $m := n - \frac{k(k+1)}{2}$ then $0 < m \leq k$, because for $m = k + 1$ we get the next triangular number $(k + 1) + \frac{k(k+1)}{2} = \frac{(k+1)(k+2)}{2}$.

We first prove that it is possible when $m < k$. We will do this by adjusting the construction for $\frac{k(k+1)}{2}$. For all $1 < i, j < k$ with $i + j = k + 1$ we can remove the stone from square (i, j) and place stones on $(i, 1)$ and $(1, j)$. This gives a valid construction with one extra stone. There are $k - 2$ such pairs (i, j) and furthermore we can always freely place a stone on $(1, 1)$, so if $m < k$ then we are done.

Then the case with $m = k$ remains. We already saw that if $k \leq 3$ it is impossible, so it remains to prove that it is possible for $k \geq 4$. We take the construction for $\frac{k(k+1)}{2} + k - 1$ as described above. From this we remove the stones on squares $(1, 1)$ and $(k - 1, k - 1)$ while adding stones on the squares $(2, 2)$, $(2, k - 1)$ and $(k - 1, 2)$. This gives a valid construction with one extra stone as long as $k - 1 > 2$ (so that you don't place multiple stones on the same square) and $(k - 1) + (k - 1) > k + 1$ (so that there was a stone on $(k - 1, k - 1)$). Both conditions are actually equivalent to $k \geq 4$, so we are done. □

4. We first note that, as quadrilateral $BGEM$ is cyclic, we have $\angle MGE = \angle MBE = 90°$. Since EG is parallel to AC, MG is therefore perpendicular to AC and therefore also to AN. Similarly, NH is perpendicular to AB and therefore to AM. In short, the intersection of MG and NH is the orthocentre of $\triangle AMN$, so these two lines are concurrent with AD if and only if AD is perpendicular to MN.



We finish the proof by proving the latter. Let P be the intersection of AD and the circumcircle of $\triangle ABC$. First assume that P is distinct from E and distinct from F. Then by alternate interior angles and using the two cyclic quadrilaterals, $\angle BME = \angle CGE = \angle BCA = \angle BPA = 180° - \angle EPB$. This implies that P lies on the circumcircle of cyclic quadrilateral $BMEG$ and similarly that P lies on the circumcircle of $CNFH$. As EM and FN are diameters of these circles, it follows that $\angle MPE = 90°$ and $\angle EPN = 90°$. From this it follows that M, P and N are collinear, and that AP is perpendicular to MN. Therefore AD is perpendicular to MN, which completes the proof in this case.

Now we consider the special case where $P = E$. Then $\angle ABP = \angle ABE = 90°$, so P is the antipode of A in the circumcircle of $\triangle ABC$. Then also $\angle ACP = 90° = \angle ACF$. As P and F both lie on AD, it follows that $P = F$. (Similarly, in the case $P = F$ it follows that $P = E$.) Furthermore, it follows that $\angle APG = \angle AEG = \angle EAC = \angle PAC = \angle PBC = \angle PBG$. So AP is the tangent line to the circumcircle of $BMPG$. Similarly, AP is the tangent line to the circumcircle of $CNPH$. However, it also holds that $MP = ME$ is the diameter of the circumcircle of cyclic quadrilateral $BMEG$ and similarly that $NP = NF$ is the diameter of the circumcircle of $CNFH$. Since these circles have a common tangent in P, these diameters lie on the same line, and AP is perpendicular to this line. □

IMO Team Selection Test 3, June 2026

Problems

1. A set S is *annual* if it consists of 2026 positive integers. Write $d(S)$ for the number of (distinct) powers of two with positive integer exponent that can be written as the sum of two distinct elements of S. Determine the maximum of $d(S)$.

2. Let $\triangle ABC$ be a triangle with $\angle BAC = 90^\circ$ and $|AB| < |AC|$. Let D, E, and F be the midpoints of BC, CA, and AB, respectively. Let P be the foot of the altitude from A on BC. A line ℓ through A intersects the interior of segment BC. Suppose that ℓ intersects the lines DE and DF in the points X and Y, respectively.

Prove that $\angle XPY = 90^\circ$ if and only if ℓ is the angular bisector of $\angle BAC$.

3. For a pair (a, b) of real numbers, we define a *move* as

- first picking four non-negative real numbers p_1, p_2, q_1, q_2 with $p_1 + p_2 = q_1 + q_2 = 1$,
- then transforming (a, b) into the pair $(p_1 b + q_2(1 - b),\ q_1 a + p_2(1 - a))$.

Suppose that we have four distinct real numbers a, b, c, d such that (a, b) can be transformed into (c, d) via a sequence of moves, and such that (c, d) can also be transformed into (a, b) via a sequence of moves. Find all possible values for $a + b + c + d$.

4. Determine all positive integers n with the following property: there exists a positive integer a such that $d + a \mid n + a$ for all positive divisors d of n.

Solutions

1. We show that the maximum is 2025. One example for which this value is attained, is given by the set S which consists of powers of two minus one: $a_i = 2^i - 1$ for $1 \leq i \leq 2026$. As $a_1 = 1$ we then see that $a_1 + a_i = 2^i$ for all $2 \leq i \leq 2026$, therefore yielding 2025 distinct powers of two.

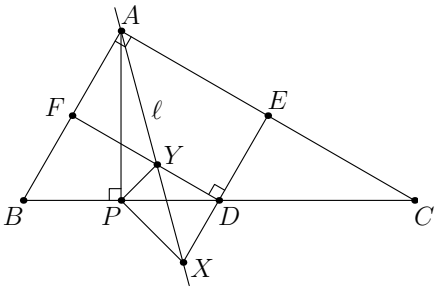
Now we show that $d(S)$ cannot be larger than 2025 for any annual set S. Given an annual set S, consider the simple graph of which the vertices are the elements of S and for which the edges are as follows: for each power of two occurring as a sum of two elements, choose a pair of elements that sum to it and add an edge between those two elements. This graph has $d(S)$ edges. It now suffices to show that the graph does not contain any cycles, as it follows from that that the graph is a forest (i.e. a disjoint union of trees) and therefore $d(S)$ is at most 2025.

Suppose for a contradiction the graph does have a cycle and let $m \geq 3$ be its length, so that we have m elements $a_1, a_2, \ldots, a_m$ and m pairwise distinct exponents $k_1, k_2, \ldots, k_m$ such that $a_i + a_{i+1} = 2^{k_i}$ for all $i \leq i \leq m$ (where $a_{m+1} = a_1$). We assume without loss of generality that k_1 is the maximum of all of the k_i. Then $\sum_{i=1}^{m} 2^{k_i} = \sum_{i=1}^{m} (a_i + a_{i+1}) = 2\sum_{i=1}^{m} a_i$ and therefore

$$2^{k_1} + a_m = a_1 + a_2 + a_m \leq \sum_{i=1}^{m} a_i = \sum_{i=1}^{m} 2^{k_i - 1}.$$

Now the sum $\sum_{i=1}^{m} 2^{k_i - 1}$ is a sum of pairwise distinct powers of two, each of which is less than 2^{k_1}. Therefore the sum itself is less than 2^{k_1}, which yields a contradiction as a_m is positive. □

2. First suppose that $\angle XPY = 90°$. If ℓ would intersect the segment BC between B and P (or at P itself), then $\angle XPY > \angle XDY = 90°$. And if ℓ would intersect the segment BC between D and C (or at D itself), then $\angle XPY < \angle XDY = 90°$. Therefore ℓ must intersect BC in the interior of the segment PD.



As $\angle XDY = 90°$ as well, it follows that $DXPY$ is cyclic. By equality of

corresponding angles, we then have

$$\angle ACP = \angle FDP = \angle YDP = \angle YXP = \angle AXP.$$

Therefore $ACXP$ is cyclic, and by Thales's Theorem, E is its circumcentre. Then, by equality of alternate interior angles and as $\triangle AXE$ is isosceles, we see that $\angle BAX = \angle EXA = \angle XAE = \angle XAC$, therefore ℓ is the angular bisector of $\angle BAC$.

For the converse, suppose that ℓ is the angular bisector of $\angle BAC$. As the angular bisector of a non-isosceles triangle through a vertex lies between the altitude and the median through that vertex, it holds that ℓ must intersect BC in the interior of the segment PD. By equality of alternate interior angles we find that $\angle AXE = \angle XAB = \angle CAX = \angle EAX$. From this, it follows that $|EX| = |EA| = |EC|$ and by Thales's Theorem this length also equals $|EP|$. Therefore $ACXP$ is a cyclic quadrilateral with circumcentre E. Hence $\angle XPD = \angle XPC = \angle XAC = \angle XYD$, so $DXPY$ is cyclic. As $\angle XDY = 90°$ it follows that $\angle XPY = 90°$. □

3. We first show that the expression $|a + b - 1|$ is a monovariant that does not increase when performing a move. We compute what happens to it during a move. For $(a', b') = (p_1 b + q_2(1 - b),\ q_1 a + p_2(1 - a))$ it holds that

$$\begin{aligned} a' + b' - 1 &= p_1 b + q_2(1 - b) + q_1 a + p_2(1 - a) - 1 \\ &= (1 - p_2)b + q_2(1 - b) + (1 - q_2)a + p_2(1 - a) - 1 \\ &= a + b - 1 + p_2(1 - a - b) + q_2(1 - a - b) \\ &= (1 - p_2 - q_2)(a + b - 1). \end{aligned}$$

We know that $p_2 + q_2 \in [0, 2]$, so the factor $1 - p_2 - q_2$ lies in the interval $[-1, 1]$. Therefore the new value $a' + b' - 1$ must lie between $-|a + b - 1|$ and $|a + b - 1|$, therefore $|a' + b' - 1| \leq |a + b - 1|$, which proves that the expression $|a + b - 1|$ is indeed a non-increasing monovariant. If $a + b \neq 1$ and $p_2 + q_2$ is not equal to 0 or 2, then performing a move strictly decreases the monovariant.

Now we return to the problem at hand. If $a + b = 1$, then the monovariant equals $|a + b - 1| = 0$, so it must remain zero after each move. Hence $c + d$ must also equal 1, therefore $a + b + c + d = 2$.

Now suppose that $a + b \neq 1$. We can transform (a, b) to (c, d) and vice versa, so in each of the moves used, the monovariant must remain constant. Therefore we can only use moves in which $p_2 + q_2$ are equal to either 0 or 2. If $p_2 + q_2 = 0$, then we must have $p_2 = q_2 = 0$, which is the move $(a, b) \to (b, a)$. If $p_2 + q_2 = 2$, then we must have $p_2 = q_2 = 1$, which is the move $(a, b) \to (1 - b, 1 - a)$. Therefore there are only four possible pairs that we can reach from (a, b):

- $(c,d) = (a,b)$: this is not possible as a, b, c, d are pairwise distinct,
- $(c,d) = (b,a)$: this is not possible as a, b, c, d are pairwise distinct,
- $(c,d) = (1-a, 1-b)$: in this case we have $a+b+c+d = 2$,
- $(c,d) = (1-b, 1-a)$: in this case we have $a+b+c+d = 2$.

It follows that $a+b+c+d$ must be equal to 2 in this case as well.

Finally, for an example that attains this value for $a+b+c+d$, consider $(a,b) = (\frac{1}{3}, \frac{1}{4})$ and $(c,d) = (\frac{3}{4}, \frac{2}{3})$. The move given by $p_2 = q_2 = 1$ then transforms these two pairs into one another. □

4. We first note that

- for $n = 1$ any a works: the only possible value for d is 1, and $1+a \mid 1+a$;
- for $n = p$ an odd prime number $a = 1$ works: the possible values for d are 1 and p, and we have $1+1 \mid p+1$ and $p+1 \mid p+1$;
- for $n = p^2$ the square of an odd prime number $a = p$ works: the possible values for d are $1, p, p^2$, and we have $1+p \mid p^2+p$, $p+p \mid p^2+p$, and $p^2+p \mid p^2+p$.

We claim there are no other n satisfying the given property.

First note that even n never satisfy the given property: if $n = 2k$, then for $d = k$ we must have $k+a \mid 2k+a$. That would imply that $k+a \mid (2k+a)-(k-a) = k$, which is not possible.

Suppose that n is odd and composite, and let p be the least prime divisor of n. We show that if a is such that $d+a \mid n+a$ for all positive divisors d of n, that then n must be p^2.

Write n as $n = pm$ where $m \geq p$. For $d = m$ we find $a+m \mid a+n$ so $a+m$ also divides $(a+n)-(a+m) = n-m = (p-1)m$. Now write $a+m = ef$, where $e \mid p-1$ and $f \mid m$. Suppose for a contradiction that $f < m$. Then $f \leq \frac{m}{p}$ (as p is the least prime divisor of n), from which follows that

$$a+m = ef \leq (p-1) \cdot \tfrac{m}{p} < m,$$

which is a contradiction. Hence $f = m$, so $a = (e-1)m$ for some divisor $e \mid p-1$.

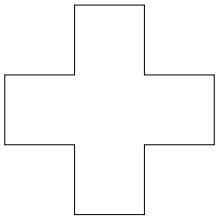
Now we find for $d = 1$ that $1+a = 1+(e-1)m \mid pm+(e-1)m$. As $1+(e-1)m$ is coprime to m, this implies that $1+(e-1)m \mid p+e-1$. But then $1+(e-1)m \leq p+e-1 \leq m+e-1$, which we can rewrite as $(e-2)(m-1) \leq 0$. As $m > 1$, it follows that $e \leq 2$, therefore that $e = 2$ as otherwise a cannot be positive. So we must have equality in $(e-2)(m-1) \leq 0$ and therefore in $p+e-1 \leq m+e-1$ as well. Hence $p = m$ and therefore $n = p^2$. □

Junior Mathematical Olympiad, June 2025

Problems

Part 1

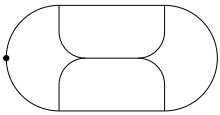
1. In the figure, all sides have the same length and all angles are right angles. The area of the figure is equal to the perimeter.



How long is each side?

A $\frac{5}{2}$ B $\frac{9}{4}$ C $\frac{11}{4}$ D $\frac{11}{5}$ E $\frac{12}{5}$

2. Every day, Birgit goes for a walk in the park. On the right you see a schematic view of the walking paths in the park. The entrance is at the dot at the left. Birgit wants to plan a walking tour. She starts at the entrance, then follows the path down first, and then walks around in such a way that she never visits the same place twice, until she gets back to the entrance.



How many different walking tours can Birgit plan like this?

A 4 B 5 C 6 D 7 E 8

3. On a 9×9 square grid, each square contains a positive integer divisible by 5. Each row contains nine different numbers and each column also contains nine different numbers. If you add up all nine numbers per row or per column, each row and each column gives the same result. If you add up all the numbers on the grid, you end up with 2025.

What is the biggest number on the grid?

A 45 B 50 C 55 D 60 E 65

4. A palindromic number is a number that does not change when you reverse the order of the digits; for example, 101 is a palindromic number, but 1231 and 1210 are not. A palindromic number may not start with a 0. Kevin calculates the number of palindromic numbers consisting of exactly 2025 digits.

How many zeros does his result end in?

A 0 B 1012 C 1013 D 2024 E 2025

5. In a country, there are five cities and each pair of cities is connected by a cycling path. Sometimes two cycling paths cross between cities, but always at

an overpass, so you can never turn off towards another city. Tom lives in one of these cities and his mother lives in one of the other four cities. Tom would like to ride a cycle route from his home that will take him first through the city where his mother lives, and then exactly once through the other cities, and finally home again. On the news in the morning, he hears that some cycling paths between the towns have been closed due to construction works, and he notices that he can now only take one possible route to complete his plan.

At least how many cycling paths are closed?

A 1 B 2 C 3 D 4 E 5

6. In a pond there are 10 lily pads in a row, numbered from 1 to 10. A frog sits on pad 1. Then it starts jumping. Each time, the frog jumps from one pad to an adjacent pad (e.g. from pad 6 it can jump to pad 5 or pad 7). After quite a few jumps, it lands on pad 1 again and stops jumping. It turns out that in total it landed exactly once on pad 1, twice on pad 2, and so on up to pad 9 where it landed exactly 9 times.

How many times did it land on pad 10?

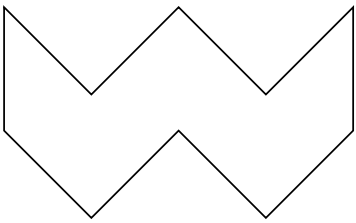
A 0 times B 1 times C 5 times D 9 times E 10 times

7. A group of neighbourhood kids are going to play football. They make a team of 6 kids with average age exactly 15 years and a team of 9 kids with average age exactly 9 years. After playing football for a while, it turns out that the older team is still too good, so they move a player from the older team to the younger team. It turns out that this raises the average age of both teams by exactly the same amount.

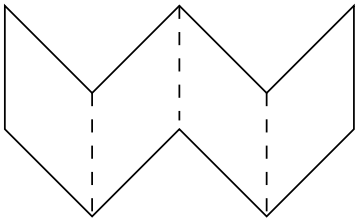
How many years old is the player who switched teams?

A 10 B 11 C 12 D 13 E 14

8. Wouter cuts out the unusual figure below of a piece of paper. All sides have the same length.



Wouter wants to divide the figure into n exactly the same pieces. If necessary, the pieces may be mirrored or rotated with respect to the other pieces. Below is an example for $n = 4$:



For how many numbers n from 2 to 8 is it possible to divide the figure into n exactly the same pieces?

A 3 B 4 C 5 D 6 E 7

Part 2

1. Aron and Nora each think of a two-digit number. It turns out that Aron's number is exactly Nora's number but with the two digits swapped. If you add Aron's and Nora's numbers, you get 165.

What number do you get if you add the two digits of Aron's number?

2. A box contains 100 chocolates. Each chocolate is made of either milk chocolate or dark chocolate. Moreover, each chocolate comes with or without nuts. In total, 26 chocolates have nuts and 54 are made of milk chocolate. The number of dark chocolates without nuts is twice the number of milk chocolates with nuts.

How many dark chocolates with nuts are there?

3. What is the smallest positive integer consisting of digits 1 to 4 in which each of the twelve digit combinations

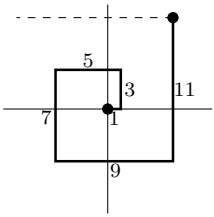
12, 13, 14, 21, 23, 24, 31, 32, 34, 41, 42, 43

occurs? (For example, in the number 14332, the number combinations 14, 43 and 32 occur.)

4. Magali has five diamonds that all have different weights. She has one diamond weighing 2 grams and one weighing 5 grams. The other three weigh more than 5 grams each. She can divide the five diamonds into two groups in such a way that the diamonds in each group weigh the same together. She can also divide them into three groups in such a way that the diamonds in each group weigh the same together.

How much do the five diamonds weigh together?

5. In the picture, an angular spiral is constructed from the origin by first drawing a line segment of length 1 to the right, then one of length 3 straight up, then one of length 5 to the left, then one of length 7 down and so on. After drawing the line segment of length 11, you end up at the point $(5, 7)$.



Which point do you end up at if you continue drawing up to and including the line segment of length 2025?

6. Ruth writes the numbers 1 to 10 side by side on a large chalkboard. She then places a plus or minus sign before each number (including before the 1) and calculates the outcome of the resulting sum. For example, $+1+2+3-4+5-6-7+8+9-10=1$.

How many possible outcomes can Ruth get?

7. A blackboard has only the number 1 written on it at first. Linda may add or multiply two numbers on the blackboard each time, and then write the result on the board. She may choose two different numbers for this, but she may also choose the same number twice. So in the first step, for example, she may write 1 $(= 1 \times 1)$ or 2 $(= 1 + 1)$ on the board.

Linda wants the number 2025 to be on the board after six steps. Give an example of the seven numbers that could then be on the board.

8. Emma sells candles online, and on her website you can order big boxes full of candles, or small boxes. There are no more than 20 candles in a box. One day, she gets a message from a customer who wants to buy exactly 25 candles, but complains that this is not possible by buying a suitable combination of big and small boxes. Emma responds that this is possible for any number of candles larger than 25.

How many candles are in a big box?

Solutions

Part 1

1. E $\frac{12}{5}$

2. D 7

3. A 45

4. B 1012

5. B 2

6. C 5 times

7. D 13

8. E 7

Part 2

1. 15

2. 6

3. 1213142324341

4. 42 gram

5. $(1013, -1012)$

6. 56

7. $1, 2, 4, 5, 9, 45, 2025$
$1, 2, 3, 5, 9, 45, 2025$
$1, 2, 3, 5, 15, 45, 2025$

8. 14

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