

# First round Dutch Mathematical Olympiad



Friday 4 February 2011

- Time available: 2 hours.
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- The answer to each B-problem is a number. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.  
NOTE: All answers should be given in exact form, like  $\frac{11}{81}$ ,  $2 + \frac{1}{2}\sqrt{5}$  or  $\frac{1}{4}\pi + 1$ .
- Formula sheets and calculators are not allowed. You can only use a pen, compass, ruler or set square and of course your mental skills.
- Good luck!

## A-problems

**A1.** The squares of a  $4 \times 4$ -field are colored black or white. The number next to each row and below each column indicates how many squares in that row or column have to be black.

In how many ways can the field be colored?

- A) 0            B) 1            C) 4            D) 5            E) 8

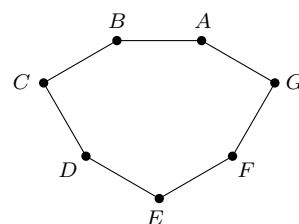
				2
				0
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				1
2	0	1	1	

**A2.** Today is 4 February 2011. This date is written down as 04-02-2011. In this problem we consider the first day from now on, of which the date is written using eight *different* digits. What is the month of that date?

- A) January    B) March    C) June    D) October    E) December

**A3.** A heptagon  $ABCDEFG$  is given, all sides of which have length 2. Moreover:  $\angle E = 120^\circ$ ,  $\angle C = \angle G = 90^\circ$  and  $\angle A = \angle B = \angle D = \angle F$ . What is the area of the heptagon?

- A)  $10 + 2\sqrt{2}$     B)  $8 + 3\sqrt{3}$     C) 14    D)  $10 + 2\sqrt{6}$     E)  $8 + 3\sqrt{6}$



**A4.** Alice, Brian and Carl have participated in a math contest consisting of 12 problems. Before the contest they were pessimistic and made the following statements.

Alice: "Brian will answer at least two more problems correctly than I will."

Brian: "I will not answer more than five problems correctly."

Carl: "I will at the most answer correctly as many problems as Alice."

Their teacher tried to encourage them by saying: "Together, you will answer more than 18 problems correctly." Afterwards, it transpired that all three students and their teacher had made a wrong prediction. Who has/have answered the least number of problems correctly?

- A) only Alice                      B) only Brian                      C) only Carl  
D) both Alice and Brian        E) you cannot be sure of that

PLEASE CONTINUE ON THE OTHER SIDE

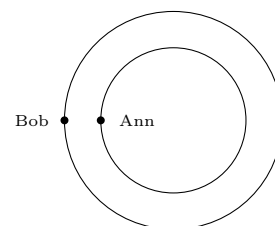
**A5.** Jack wants to write down some of the numbers from 1 to 100 on a piece of paper. He wants to do it in such a way, that no two numbers on the piece of paper will add up to 125. How many numbers, at most, can Jack write down on his piece of paper?

- A) 50      B) 61      C) 62      D) 63      E) 64

**A6.** The number  $a = 11\dots 111$  consists of 2011 digits '1'. What is the remainder of  $a$  when divided by 37?

- A) 0      B) 1      C) 3      D) 7      E) 11

**A7.** Ann and Bob are sitting in a fairground attraction. They move in circles around the same center and in the same direction. Ann moves around once every 20 seconds, Bob once every 28 seconds. At a certain moment they are at minimum distance from each other (see figure).



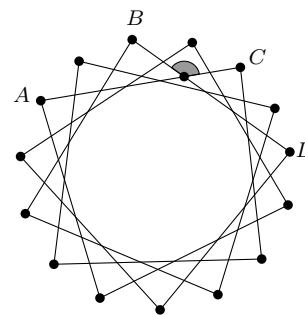
How many seconds does it take, from that moment on, until Ann and Bob are at maximum distance from each other?

- A) 22.5      B) 35      C) 40      D) 49      E) 70

**A8.** The vertices of a regular 15-gon are connected as in the figure. (Mind you: the sizes in the figure are not entirely correct!)

What is the size of the angle, indicated by the arc, between  $AC$  and  $BD$ ?

- A)  $130^\circ$       B)  $132^\circ$       C)  $135^\circ$       D)  $136^\circ$       E)  $137.5^\circ$



## B-problems

**B1.** A number  $x$  satisfies:  $x = \frac{1}{1+x}$ . Determine  $x - \frac{1}{x}$ . Simplify your answer as much as possible.

**B2.** An escalator goes up from the first to the second floor of a department store. Dion, while going up the escalator, also walks at a constant pace. Raymond, going in the opposite direction, tries to walk downwards, from the second to the first floor, on the same escalator. He walks at the same pace as Dion. They both take one step of the escalator at a time. Dion arrives at the second floor after exactly 12 steps; Raymond arrives at the first floor after exactly 60 steps. How many steps would it take Dion to get upstairs if the escalator would stand still?

**B3.** Six scouts are on a scouting expedition. They are going to the woods on Saturday, and into the mountains on Sunday. On both days, they have to go in pairs. The scoutmaster wants to group them into pairs for both expeditions in such a way that nobody has the same partner on the second day as on the first day. In how many ways can he do that?

**B4.** In the figure you see a pointed arch  $ABC$  and its inscribed circle. The pointed arch consists of a line segment  $AB$  of length 1, a circular arc  $BC$  with center  $A$  and a circular arc  $AC$  with center  $B$ .

What is the radius of the inscribed circle of this pointed arch?

