Second Round
Dutch Mathematical Olympiad

NEDERLANDSE
WISKUNDE
OLYMPIADE

Friday, March 26, 2010

- Time available: 2.5 hours.
- The competition consists of five B-problems and two C-problems.
- Formula sheets and calculators are not allowed. You can only use a pen, compass, ruler or set square and of course your mental skills.
- Good luck!

B-problems

The answer to each B-problem is a number. A correct answer is awarded 4 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
NOTE: all answers should be given in exact form, like $\frac{11}{81}$ or 5^8 or $\frac{1}{4}(\sqrt{5} + \pi)$.

B1. Alice has got five real numbers $a < b < c < d < e$. She takes the sum of each pair of numbers and writes down the ten sums. The three smallest sums are 32, 36 and 37, while the two largest sums are 48 and 51. Determine e.

B2. Let AB be a diameter of a circle. Point C lies on line segment AB such that $2 \cdot |AC| = |BC|$. The points D and E lie on the circle such that CD is perpendicular to AB and such that DE is also a diameter of the circle. Write the areas of the triangles ABD and CDE as $O(ABD)$ and $O(CDE)$. Determine the value of $\frac{O(ABD)}{O(CDE)}$.

B3. A 24-hour digital clock displays the times from 00:00:00 till 23:59:59 during the day. You can add the digits of the time on every second of the day; this will give you an integer. For example, at 13:07:14 you will get $1 + 3 + 0 + 7 + 1 + 4 = 16$. When you write down this sum for every possible state of the clock and then take the average of all these numbers, what will be the result?

B4. For the infinite sequence of numbers

$$0, \ 1, \ 2, \ 2, \ 1, -1, -2, -1, \ 1, \ 3, \ \ldots$$

the following rule holds. For each four consecutive numbers $\ldots, a, b, c, d, \ldots$ of the sequence the number d is equal to c minus the smallest of the two numbers a and b. For example, the ninth number of the sequence is equal to $-1 - (-2) = 1$ and the tenth number is equal to $1 - (-2) = 3$. Calculate the 100th number of this sequence.

B5. Raymond has got five coins. On the heads side of each coin is the number 1. On the tails sides of the coins are the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$ respectively. Because every coin has got either its heads side or its tails side facing up, there are 32 ways to put the five coins on the table. Raymond multiplies the five numbers facing up for each of these 32 situations and writes down all results.
If Raymond adds up these 32 numbers, what will be the final result?

C-problems

For the C-problems not only the answer is important; you also have to describe the way you solved the problem. Use separate sheets of paper for each C-problem. A correct and well-explained answer is awarded 10 points.

Partial solutions may also be worth some points. Therefore, write neatly and hand in your drafts (for each problem).

C1. Determine all positive integers n consisting of four digits for which it holds that n plus the sum of the digits of n is equal to 2010.

C2. Line segment AB has length 10. A point C lies on line segment AB such that $|AC| = 6$ and $|CB| = 4$. Two points X and Y lie on the same side of the line AB, such that $|YB| = |YC| = 3$, $|XA| = 8$ and $|XC| = 6$. Determine the length of line segment XY.