1. Let $a$, $b$, $c$, and $d$ be four distinct integers. Prove that $(a - b)(a - c)(a - d)(b - c)(b - d)(c - d)$ is divisible by 12.

2. We number the columns of an $n \times n$-board from 1 to $n$. In each cell, we place a number. This is done in such a way that each row precisely contains the numbers 1 to $n$ (in some order), and also each column contains the numbers 1 to $n$ (in some order). Next, each cell that contains a number greater than the cell’s column number, is coloured blue. In the figure below you can see an example for the case $n = 3$.

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1 2 3
3 1 2
1 2 3
2 3 1
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(a) Suppose that $n = 5$. Can the numbers be placed in such a way that each row contains the same number of blue cells?

(b) Suppose that $n = 10$. Can the numbers be placed in such a way that each row contains the same number of blue cells?

3. Determine all pairs $(p, m)$ consisting of a prime number $p$ and a positive integer $m$, for which

$$p^3 + m(p + 2) = m^2 + p + 1$$

holds.
4. We are given an acute triangle $ABC$ and points $D$ on $BC$ and $E$ on $AC$ such that $AD$ is perpendicular to $BC$ and $BE$ is perpendicular to $AC$. The intersection of $AD$ and $BE$ is called $H$. A line through $H$ intersects line segment $BC$ in $P$, and intersects line segment $AC$ in $Q$. Furthermore, $K$ is a point on $BE$ such that $PK$ is perpendicular to $BE$, and $L$ is a point on $AD$ such that $QL$ is perpendicular to $AD$.

Prove that $DK$ and $EL$ are parallel.

5. The numbers 1 to 12 are arranged in a sequence. The number of ways this can be done equals $12 \times 11 \times 10 \times \cdots \times 1$. We impose the condition that in the sequence there should be exactly one number that is smaller than the number directly preceding it. How many of the $12 \times 11 \times 10 \times \cdots \times 1$ sequences meet this demand?