First round
Dutch Mathematical Olympiad

21 January – 31 January 2013

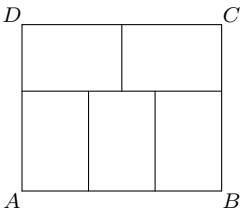
- Time available: 2 hours.
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- The answer to each B-problem is a number or multiple numbers. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
  NOTE: All answers should be given in exact form, like $\frac{11}{81}$, $2+\frac{1}{2}\sqrt{5}$ or $\frac{1}{4}\pi+1$.
- Formula sheets and calculators are not allowed. You can only use a pen, compass, ruler or set square and of course your mental skills.
- After the contest, hand in your answer sheet, this problem sheet and any scrap paper. The problems and solutions will be available from 1 February on the website: `www.wiskundeolympiade.nl`.
- Good luck!

A-problems

A1. A traffic light is alternately green and red. The periods green and red are equally long and always of the same length: either 1, 2, or 3 minutes. There are four combinations for the colour of the light at 12:08 pm and at 12:09 pm: red–red, red–green, green–red, and green–green.
How many of these four combinations are possible, given that the light is red at 12:05 pm and also red at 12:12 pm?

A) 1 B) 2 C) 3
D) 4 E) The light cannot be red at both 12:05 pm and 12:12 pm.

A2. The rectangle $ABCD$ is divided into five equal rectangles. The perimeter of each of these small rectangles is 20.
What is the area of rectangle $ABCD$?



A) 72 B) 112 C) 120 D) 140 E) 150

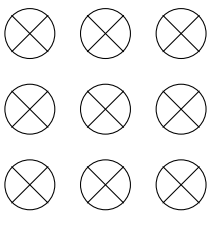
A3. The numbers a, b, c, d and e satisfy:

$$a+b+1 \;=\; b+c-2 \;=\; c+d+3 \;=\; d+e-4 \;=\; e+a+5.$$

Which is the largest of these five numbers?

A) a B) b C) c D) d E) e

A4. Nine light bulbs are put in a square formation. Each bulb can be either *on* or *off*. We can make a move by pressing a bulb. Then, the pressed bulb and the bulbs in the same row or column change their state from *on* to *off* or vice versa. Initially, all light bulbs are *on*.
What is the minimum number of moves needed to turn *off* all the light bulbs?



A) 3 B) 4 C) 5 D) 9 E) This is impossible.

PLEASE CONTINUE ON THE OTHER SIDE

A5. Out of a shipment of boxes, one fourth is empty. We open one fourth of all boxes and notice that one fifth of them is non-empty.
Which part of the unopened boxes is empty?

A) $\frac{4}{15}$ B) $\frac{1}{4}$ C) $\frac{1}{15}$ D) $\frac{1}{16}$ E) $\frac{1}{20}$

A6. A regular hexagon and an equilateral triangle have the same perimeter.
What is the ratio *area hexagon* : *area triangle*?

A) 2 : 3 B) 1 : 1 C) 4 : 3 D) 3 : 2 E) 2 : 1

A7. What are the last four digits of 5^{2013}?

A) 0625 B) 2525 C) 3125 D) 5625 E) 8125

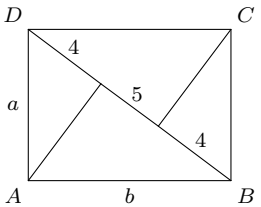
A8. Twenty students did a test. No two students answered the same number of questions correctly. Each question was answered correctly by at most three students.
What is the smallest number of questions that the test could have had?

A) 63 B) 64 C) 67 D) 70 E) 71

B-problems

B1. What is the smallest positive integer consisting of the digits 2, 4 and 8, such that each digit occurs at least twice and the number is not divisible by 4?

B2. A rectangle $ABCD$ has sides of length a and b, where $a < b$. The lines through A and C perpendicular to the diagonal BD divide the diagonal into three segments of lengths 4, 5, and 4.
Calculate $\frac{b}{a}$.



B3. A bus calls at three stops. The middle bus stop is equally far from the first stop as from the last stop. Fred, standing at the middle bus stop, has to wait for 15 minutes for the bus to arrive. If he cycles to the first stop, he will arrive there at the same time as the bus. If instead he runs to the last stop, he will also arrive there at the same moment as the bus.
How long would it take Fred to cycle to the last stop and then run back to the middle stop?

B4. We write down the numbers from 1 to 30 000 one after the other to form a long string of digits:

$$123456789101112\ldots30000.$$

How many times does 2013 occur in this sequence?