Junior Wiskunde Olympiade
Problems part 1

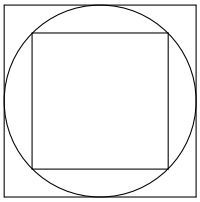
Saturday 3 October 2015
Vrije Universiteit Amsterdam

- The problems in part 1 are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form.
- A correct answer is awarded 2 points, for a wrong answer no points are given.
- You are allowed to use draft paper. The use of compass, ruler or set square is allowed. Calculators and comparable devices are not allowed.
- You have 45 minutes to finish these problems. **Good luck!**

1. A booklet is made by forming a stack of 11 sheets of paper and then folding the stack in half. The pages of the booklet are numbered, like in a book, from 1 to 44, where the front cover gets the number 1 and the back cover gets number 44. Now the booklet is opened up and from the stack of 11 sheets we take the one in the middle.
Adding up the four numbers on this sheet, what outcome do we get?

A) 82 B) 84 C) 86 D) 88 E) 90

2. We draw a circle through the four vertices of a square of area 1. Then, we draw a square around this circle in such a way that that all the sides are tangent to the circle.
What is the area of this square?



A) $\frac{10}{7}$ B) $\frac{3}{2}$ C) $\frac{5}{3}$ D) 2 E) $\frac{100}{49}$

3. Quintijn has three equally big and equally filled bottles of wine. Bottles 1 and 3 contain the same kind of white wine, while bottle 2 contains red wine. Quintijn now pours a small amount of wine from bottle 1 into bottle 2. Next, after mixing the content of bottle 2 really well, he pours the same amount from bottle 2 into bottle 3. In the same way, he pours the same amount of wine from bottle 3 into bottle 1. Now all bottles contain the same amount of wine as they did at the start. However, the content of each bottle is polluted with wine of the other type.
Which bottle is polluted most?

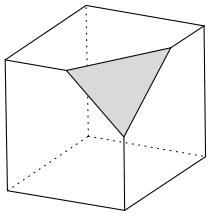
A) bottle 1 B) bottle 2 C) bottle 3
D) all three equally E) you cannot determine this

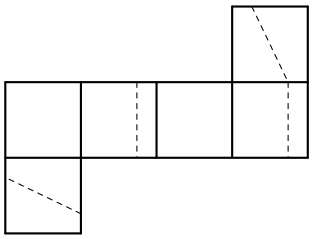
4. Aad, Bep, Cor, Dirk, Eva, and Fenna are sitting in this order in a circle around the campfire. Aad has a torch. He gives it to Bep who is sitting one place to his right. She gives the torch to Dirk, who is sitting two places to her right. He gives the torch to Aad, who is sitting three places to his right, et cetera. It happens that someone must give the torch to the person sitting, for instance, six or twelve positions to the right. Then this person gives the torch to him- or herself.
When Dirk is given the torch for the hundredth time, who does he pass the torch to?

A) Aad B) Bep C) Cor D) Dirk E) Eva

PLEASE CONTINUE ON THE OTHER SIDE

5. When a cube is cut with a plane, a cross section is created. This figure is formed by the lines where the plane cuts the facets of the cube. In the left figure, you can see an example in which the cross section is a triangle.





In the right figure, a flattened cube is drawn with the cut lines drawn on the facets.
What is the cross section of the cube corresponding to this figure?

A) a triangle B) a square C) a rectangle, but not a square
D) a hexagon E) a parallelogram, but not a rectangle

6. On a machine there are three buttons. The first button can be used to add 20 marbles to a tray in the machine. The second button can be used to increase the number of marbles in the tray by 20%, after which 15 additional marbles are added. The third button can be used to increase the number of marbles in the tray by 50%. If pressing a button would cause the number of marbles to be non-integral, the pressing of the button is not allowed. In the beginning the tray is empty. After some button presses, there are 91 marbles in the tray.
How often has the first button been pressed?

A) 0 B) 1 C) 2 D) 3 E) 4

7. Ria, Sophie, and Tine are sitting around a round table in clockwise order and are playing a game with chips. Ria starts with 3 chips, Sophie with 4 chips, and Tine with 5 chips. In each round they simultaneously give chips to one of their neighbours. Each player can choose to give 2 chips to her right neighbour or 1 chip to her left neighbour. If someone has no more chips, the game ends.
Is it possible for the players to have the same number of chips after some number of rounds, and if this is the case, how many rounds have to be played at least to accomplish this?

A) no, it is impossible. B) yes, 3 rounds. C) yes, 6 rounds.
D) yes, 7 rounds. E) yes, 8 rounds.

8. Six people are sitting around a round table. Each of them is either a knight or a knave. Knights always speak the truth, while knaves always lie. Each of them has a card containing a number. All numbers are different and everyone knows the numbers of their two neighbours. When asked: "Is your number greater than the numbers of both your neighbours?", everyone answers with "Yes". When asked: "Is your number smaller than the numbers of both your neighbours?", at least one person answers "Yes" and at least one answers "No".
What are the possible numbers of people answering "Yes" to this second question?

A) 1 or 2 B) 1 or 3 C) 2 or 3 D) 2 or 4 E) 2, 3, or 4