Second round
Dutch Mathematical Olympiad

Friday 14 March 2014

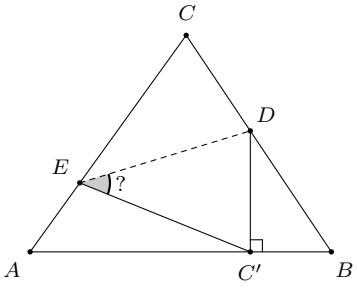
- Time available: 2.5 hours.
- The competition consists of five B-problems and two C-problems.
- Formula sheets and calculators are not allowed. You can only use a pen, compass, ruler or set square and of course your mental skills.
- Good luck!

B-problems

The answer to each B-problem is a number. A correct answer is awarded 4 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer. NOTE: all answers should be given in exact form, like $\frac{11}{81}$ or 5^8 or $\frac{1}{4}(\sqrt{5}+\pi)$.

B1. Brenda is filling pouches from an unlimited supply of red and blue marbles. In each pouch she puts more red than blue marbles, and each pouch can contain at most 25 marbles. For example, she can make a pouch containing 6 red marbles and 2 blue marbles, or a pouch containing 1 red marble and 0 blue marbles.
How many differently filled pouches can she make in total?

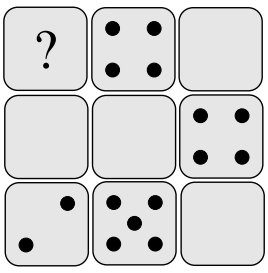
B2. In the figure an equilateral triangle ABC is drawn with points D and E on sides BC and AC. When folding the triangle along the line DE, the vertex C is folded onto point C' on line AB. Furthermore, $\angle DC'B = 90°$ holds.
What is the size of $\angle DEC'$?
Beware: the figure is not drawn to scale.



B3. For how many of the integers n from 1 up to and including 100 is the number $8n+1$ a perfect square?

B4. Evan and nine other people are standing in a circle. All ten of them think of an integer (that may be negative) and whisper their number to both of their neighbours. Afterwards, they all state the average of the two numbers that were whispered in their ear. Evan states the number 10, his right neighbour states the number 9, the next person along the circle states the number 8, and so on, finishing with Evan's left neighbour who states the number 1.
What number did Evan have in mind?

B5. The numbers of dots on two opposite faces of a die always sum to 7. Nine identical dice are glued in a 3×3-array. This is done in such a way that when two faces are glued together, they must contain the same number of dots. In the figure you can see the top view of the array. For five of the dice the number of dots is not shown.
What number of dots must be on the place of the question mark?



PLEASE CONTINUE ON THE OTHER SIDE

C-problems

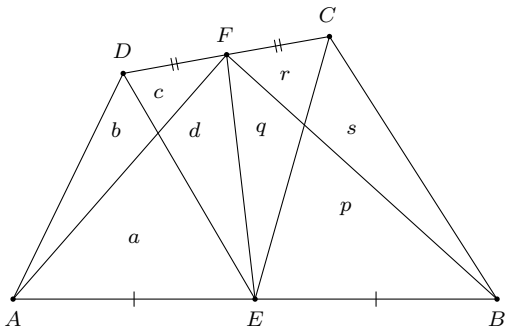
For the C-problems not only the answer is important; you also have to write down a clear reasoning. Use separate sheets of paper for each C-problem. A correct and well-explained answer is awarded 10 points.
Partial solutions may also be worth some points. Therefore, write neatly and hand in your drafts (for each problem separately).

C1. We are given a quadrilateral $ABCD$. The midpoint of AB is denoted by E and the midpoint of CD is denoted by F. The segments AF, BF, CE, DE, and EF divide the quadrilateral into eight triangles. The areas of these triangles are denoted by the letters from a to d and p to s, see the figure.

(a) Prove that $a + d = p + q$.

(b) Prove that $a + r = c + p$.

(c) Prove that $b + s = d + q$.



C2. A positive integer n is called a *jackpot number* if it has the following property: there exists a positive integer k consisting of two or more digits, all of which are equal (such as 11111 or 888), for which the product $n \cdot k$ is again a number consisting of equal digits. For example, 3 is a jackpot number because $3 \cdot 222 = 666$.

(a) Find a jackpot number consisting of 10 digits and prove that it is a jackpot number.

(b) Prove that 11 is not a jackpot number.

(c) Determine whether 143 is a jackpot number and prove that your answer is correct.