

# First round

## Dutch Mathematical Olympiad



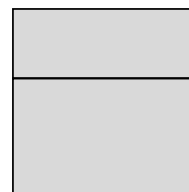
19 January – 29 January 2015

- Time available: 2 hours.
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- The answer to each B-problem is a number or multiple numbers. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.  
NOTE: All answers should be given in exact form, like  $\frac{11}{81}$ ,  $2 + \frac{1}{2}\sqrt{5}$  or  $\frac{1}{4}\pi + 1$ .
- Formula sheets and calculators are not allowed. You can only use a pen, paper, compass, ruler or set square and of course your mental skills.
- After the contest, hand in your answer sheet, this problem sheet and any scrap paper. The problems and solutions will be available from 30 January on the website: [www.wiskundeolympiade.nl](http://www.wiskundeolympiade.nl).
- Good luck!

### A-problems

1. A square is divided into two rectangular pieces by a straight line. The sum of the circumferences of the two rectangles is 30 centimetres. What is the side length of the square in centimetres?

A) 5      B) 6      C)  $\frac{15}{2}$       D) 8      E) 12



2. Five suspects are being questioned about the order of arrival at a crime scene. They make the following statements.

**Aad:** "I arrived first."

**Bas:** "I arrived second."

**Carl:** "I arrived third."

**Dave:** "Of Aad and Bas, one arrived before me and the other arrived after me."

**Erik:** "Of Bas and Carl, one arrived before me and the other arrived after me."

It is known that exactly one of the suspects lied.

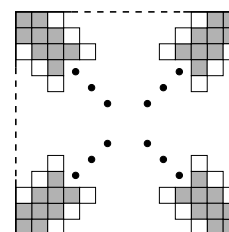
Who was the fourth to arrive at the crime scene?

A) Aad      B) Bas      C) Carl      D) Dave      E) Erik

3. A big square consists of 2015 times 2015 small squares. The small squares on the two main diagonals and on the four adjacent diagonals are coloured grey, and the rest is coloured white (see the figure).

How many small squares are coloured grey?

A) 12081      B) 12082      C) 12085      D) 12086      E) 12090



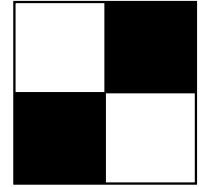
PLEASE CONTINUE ON THE OTHER SIDE

4. The difference of two integers is 10. If you multiply the two integers, you will get one of the following five numbers.

Which number do you get?

- A) 22398    B) 22420    C) 22442    D) 22453    E) 22475

5. Jan has got a wooden cube. He divides each of the faces into  $2 \times 2$  squares that he subsequently paints in a black-white pattern: two diagonally opposite squares are painted black, the other two are painted white. In each vertex of the cube three squares meet. If two or three of these squares are black, we call the vertex a *dark* vertex.



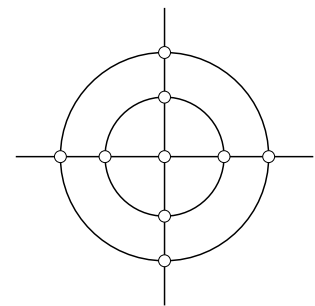
What is the smallest number of dark vertices that the cube can have?

- A) 0    B) 1    C) 2    D) 3    E) 4

6. In how many ways can you get the number 100 by adding some consecutive integers between 1 and 99 inclusive?

- A) 1    B) 2    C) 3    D) 4    E) 5

7. In the figure, you see two circles and two lines together with the nine nodes in which they intersect. Jaap wants to colour exactly four of the nodes red, in such a way that no three red nodes are on the same line or on the same circle.



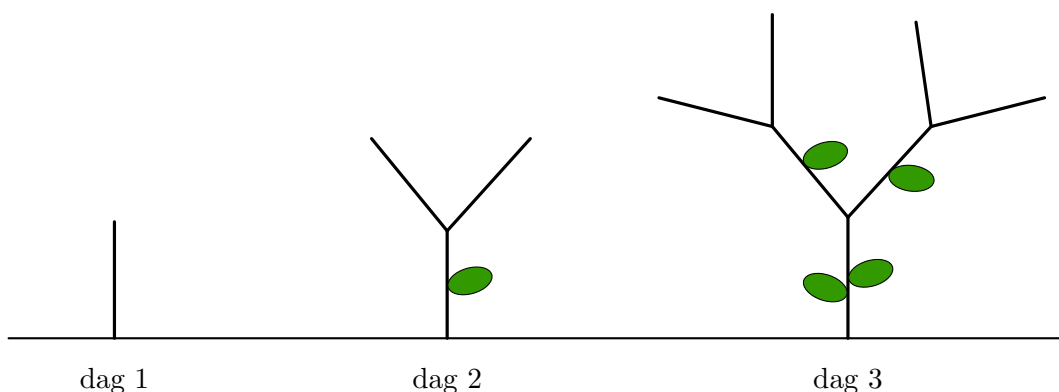
How many such colourings can Jaap make?

- A) 6    B) 12    C) 18    D) 24    E) 36

8. A tree grows in the following manner. On day 1, one branch grows out of the ground. On day 2, a leaf grows on the branch and the branch tip splits up into two new branches. On each subsequent day, a new leaf grows on every existing branch and each branch tip splits up into two new branches. See the figure below.

How many leaves does the tree have at the end of the tenth day?

- A) 172    B) 503    C) 920    D) 1013    E) 2047



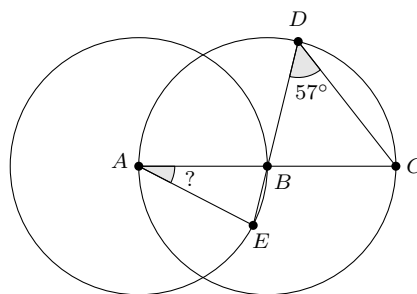
## B-problems

1. Julia constructs a sequence of numbers. She starts with two integers she chooses herself. Then, she calculates the next numbers in the sequence as follows: if the last number she wrote down is  $b$  and the number before that is  $a$ , then the next number will be  $2b - a$ . The second number in Julia's sequence is 55 and the hundredth number is 2015.

What is the first number in Julia's sequence?

2. Two points  $A$  and  $B$  and two circles are given, one having  $A$  as centre and going through  $B$  and the other one having  $B$  as centre and going through  $A$ . Point  $C$  lies on the second circle and on line  $AB$ . Point  $D$  also lies on the second circle. Point  $E$  lies on the first circle and on line  $BD$ . See the figure below. Moreover,  $\angle D = 57^\circ$ .

What is the value of  $\angle A$  in degrees?



3. A positive integer is called *alternating* if its digits alternate between even and odd. For example, 2381 and 3218 are alternating, but 2318 is not. An integer is called *super alternating* if the number itself is alternating and twice that number is alternating as well. For example, 505 is super alternating, because both 505 and 1010 are alternating.

How many super alternating integers consisting of four digits exist?

Pay attention: a four digit integer cannot start with a 0.

4. On a school trip, twenty students will be abseiling. In each round, one student will descend the mountain. Hence, after twenty rounds, all students will have gone down the mountain safely. In the first round, cards bearing the numbers 1 to 20 are distributed among the students. The student getting number 1 will go down first. In round 2, cards bearing the numbers 1 to 19 are distributed among the remaining students. The student receiving the number 1 is next to descend. They continue in this way, until there is only one student left in round 20, who automatically gets a card bearing the number 1. By an amazing coincidence, no student gets the same number twice. In the first round, Sara gets a card with number 11.

What is the sum of the numbers on the cards received by Sara?