First round
Dutch Mathematical Olympiad

20 January – 30 January 2014

- Time available: 2 hours.
- The A-problems are multiple choice questions. Exactly one of the five given options is correct. Please circle the letter of the correct answer on the form. A correct answer is awarded 2 points, for a wrong answer no points are given.
- The answer to each B-problem is a number or multiple numbers. A correct answer is awarded 5 points, for a wrong answer no points are given. Please work very accurately: a minor error in a calculation may result in a wrong answer.
  NOTE: All answers should be given in exact form, like $\frac{11}{81}$, $2 + \frac{1}{2}\sqrt{5}$ or $\frac{1}{4}\pi + 1$.
- Formula sheets and calculators are not allowed. You can only use a pen, compass, ruler or set square and of course your mental skills.
- After the contest, hand in your answer sheet, this problem sheet and any scrap paper. The problems and solutions will be available from 31 January on the website: `www.wiskundeolympiade.nl`.
- Good luck!

A-problems

1. We are given a 4×4 table and want to colour four of the 16 cells black. This should be done in such a way that every row and every column has exactly one black cell, and no two black cells are diagonally adjacent (share a corner point).
   In how many ways can we choose the four black cells?

   A) 1   B) 2   C) 3   D) 4   E) It is impossible.

2. A pond contains both red and yellow carp. Two fifths of the carp are yellow, the rest of the carp are red. Three quarters of the yellow carp are female. In total, there are an equal number of male and female carp.
   Which fraction of the total carp population are red males?

   A) $\frac{1}{5}$ B) $\frac{1}{4}$ C) $\frac{3}{10}$ D) $\frac{2}{5}$ E) $\frac{1}{2}$

3. Seven lily pads are numbered 1 through 7 from left to right. A frog jumps along these pads. It can jump to the left and to the right, but only by leaps of three or five pads at once. For example, starting from pad 2, it can only leap to pads 5 and 7. The frog wants to make a journey in which he visits each pad exactly once (so the first and last pad on his journey will be different).
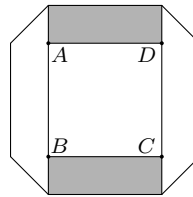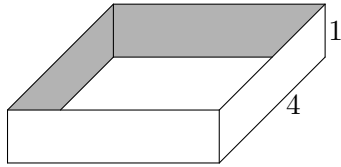
   ① ② ③ ④ ⑤ ⑥ ⑦

   Which pads can be the starting point of such a journey?

   A) pads 1 to 7   B) pads 1, 3, 5, and 7   C) pads 3 and 5
   D) pad 4   E) none of the pads

4. A square paper ring has height 1, and the sides have length 4. The ring is depicted in the left hand figure. By folding it flat on the tabletop, we get the right hand figure, where $ABCD$ is a square.
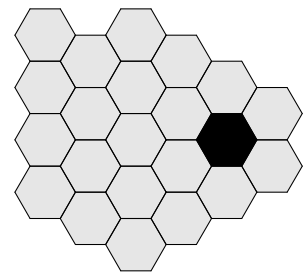

What is the length of side AB?

A) $\frac{5}{2}$ B) 3   C) $\frac{7}{2}$ D) 4   E) $\frac{9}{2}$

5. Tom and Jerry were running a race. The number of runners finishing before Tom was equal to the number of runners finishing after him. The number of runners finishing before Jerry was three times the number of runners finishing after him. In the final ranking, there are precisely 10 runners in between Tom and Jerry. All runners finished the race, and no two runners finished at the same time.
How many runners participated in the race?

A) 22   B) 23   C) 41   D) 43   E) 45

6. A garden with a pond (the black hexagon) will be tiled using hexagonal tiles as in the figure. The tiles come in three colours: red, green and blue. No two tiles that share a side can be of the same colour.
In how many ways can the garden be tiled?

A) 3   B) 6   C) 12   D) 18   E) 24

7. In the figure on the right, a quadrilateral $ABCD$ is drawn. The midpoint of side AB is called M. The four line segments AM, BM, BC, and AD each have length 8, and the line segments DM and CM both have length 5. What is the length of line segment CD?
Beware: the figure is not drawn to scale.
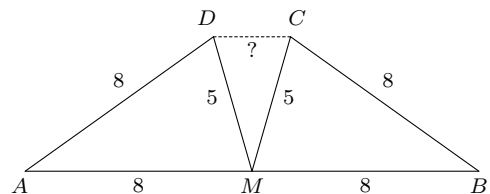
A) 3   B) $\frac{40}{13}$ C) $\frac{25}{8}$ D) $\frac{16}{5}$ E) $\frac{13}{4}$

8. A motorboat is moving with a speed of 25 kilometres per hour, relative to the water. It is going from Arnhem to Zwolle, moving with the constant current. At a certain moment, it has travelled 42% of the total distance. From that point on, it takes the same amount of time to reach Zwolle as it would to travel back to Arnhem.
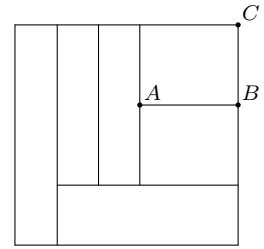What is the speed of the current (in kilometres per hour)?

A) 3   B) 4   C) $\frac{9}{2}$ D) 5   E) 6

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B-problems

1. A square is divided into six rectangles, all of the same area. The length of side AB equals 5.
What is the length of side BC?
Beware: the figure is not drawn to scale.

2. Carl has a large number of apples and pears. He wants to pick ten pieces of fruit and place them in a row. He wants to do it in such a way that there is no pear anywhere between two apples. For example, the fruit sequences AAAAAAAAAA and AAPPPPPPPP are allowed, but AAPPPPPPPA and APPPPPPPAA are not.
How many sequences can Carl make?

3. If you were to compute

$$\underbrace{999\ldots99}_{\text{2014 nines}} \quad \times \quad \underbrace{444\ldots44}_{\text{2014 fours}}$$

and then add up all digits of the resulting number, what number would the final outcome be?

4. We consider 5×5-tables containing a number in each of the 25 cells. The same number may occur in different cells, but no row or column contains five equal numbers. Such a table is called *pretty* if in each row the cell in the middle contains the average of the numbers in that row, and in each column the cell in the middle contains the average of the numbers in that column. The *score* of a pretty table is the number of cells that contain a number that is smaller than the number in the cell in the very middle of the table.
What is the smallest possible score of a pretty table?